# 3. Force and Motion

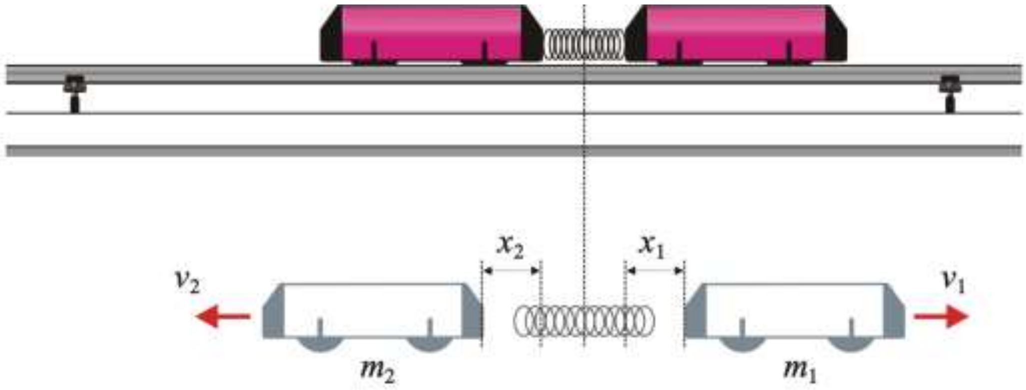
# 3.1

# Theory

Dynamics is a theory that explains an object's motion when a force that depends on the location or velocity is working, and this can be explained with Newtonian mechanics. Chapter 3 deals with the velocity, momentum, acceleration and energy of an object in Newtonian mechanics. In a plain motion which conserves the momentum and the energy[1], Newton's laws of inertia, force, acceleration and action-reaction can be explained. Also, to understand physical phenomena generally, physical situations concerning force and motion will be expressed with Lagrange's equation of motion[2].

When expressing the object's motion, the reference frame is needed, which helps to express it with the spectator's location as a standard. Chapter 3 deals with the first dimension motion of straight line and it uses Cartesian coordinate. Also, this chapter contains the theories and experiments about the location, velocity and energy of an object's motion.

[1] When an object gets a force depending on the velocity and moves, the law of action and reaction cannot be applied in this case. Chapter 3 deals with a cart's motion when the damping resistance in proportion to the velocity is working.
[2] Force and motion can be expressed with Newtonian mechanics and Lagrangian mechanics.




Picture 3.1.1 two carts' motion with constant force: if you release the carts slightly when the spring is compressed, they both move with their own velocities.

### 3.1.1. Newtonian Mechanics

As in picture 3.1.1, when the two carts moves away from each other on the first dimension straight track because of the spring's force, the relationships between the carts' masses[3] and velocity $v_1 = \dot{x}_1$, and acceleration $a_1 = \ddot{x}_1$ are as follows.

$$m_1 v_1 = - m_2 v_2$$

$$m_1 \frac{dv_1}{dt} = - m_2 \frac{dv_2}{dt}$$

The multiplication between mass and velocity becomes linear momentum[4] and the object's change of motion expressed in Newton's second law is represented as the result of linear momentum's change which changes as time goes by. This changing rate is in proportion to the force acted upon the object and can be expressed[5] as below.

$$F = m\frac{dv}{dt} = ma \qquad (3.1.1)$$

[3] Mass is the quantitative value of inertia that can be measured within the inertial frame that conserves momentum and energy.
[4] When the space has homogeneity, the moving object's linear momentum is conserved, and this shows that Newton's third law, the law of action and reaction means the law of conservation of momentum.
[5] When a constant force $F$ is given, it is the dynamical solution of a moving object that has acceleration $a$.

This formula can be rewritten like below with momentum $P = mv$ .

$$F = m\frac{dv}{dt} = \frac{dP}{dt} \qquad (3.1.2)$$

So, the changing rate of two carts' total momentum[6] according to time is as follows.

$$\frac{d}{dt}(P_1 + P_2) = 0$$

To find out the location, velocity and acceleration of an object in the inertial frame in which the force is regularly working, you can solve the formula 3.1.1 as below.

$$mdv = Fdt$$

$$\int_{v_0}^{v} dv = \frac{F}{m}\int_{t_0}^{t} dt$$

$$v - v_0 = \frac{F}{m}(t - t_0)$$

When $F = ma, t_0 = 0$ , the formula about the velocity $v$ can be changed from the formula above to below.

$$v = v_0 + at \qquad (3.1.3)$$

From this formula, do the integral calculus to both sides and solve it for the object's location $x$.

$$\frac{dx}{dt} = v_0 + at$$

$$\int_{x_0}^{x} dx = \int_{0}^{t} v_0 + at$$

$$x = x_0 + v_0 t + \frac{1}{2}at^2$$

[6] More detailed information about momentum will be included in chapter 4. Collision.

Here, if the object's initial location is $x_0 = 0$ , the result is as follows.

$$x = v_0 t + \frac{1}{2}at^2 \qquad (3.1.4)$$

Nest, let's find out about the object's energy. When a constant force is working, the location and velocity of the object is as formula (3.1.3), (3.1.4), and if this force[7] is defined as the function of location only, it is like below.

$$F(x) = -\nabla V(x) \qquad (3.1.5)$$

The object's total energy $E$ can be expressed as the sum of the kinetic energy[8] $T$ and potential energy $V$.

$$E = \frac{1}{2}mv^2 + V(x) \qquad (3.1.6)$$

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### Exercise 3.1.1: The Energy of Moving Object When the Conservative Force Is Working

1. Calculate the energy of an object in free fall motion within the central field[9].

2. In picture 3.3.1 calculate the cart's potential energy when it receives force $F = \mathbf{l}kx$ by the spring.

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[7] When the force is defined as the function of location only, it is called conservative force.

[8] $F = m\ddot{x} = m\dot{x}\left(\frac{d\dot{x}}{dx}\right) = \frac{d}{dx\left(\frac{1}{2}m\dot{x}^2\right)}$ and if $T = \frac{1}{2}mv^2$ , $F = \frac{dT}{dx}$.

[9] This is the case that the potential energy is expressed as the function of distance only from the center. Electromagnetic field or gravitational field is the case that the potential energy is in inverse proportion to the distance and the force in this case is in inverse proportion to the square of distance.

Explanation 1:

In formula (3.1.5), if the gravity $F = -mg$ [10] , the result is as follows.

$$F = -mg = -\frac{dV}{dy}$$

So, in formula (3.1.6), the object's energy can be calculated like below.

$$E = \frac{1}{2}mv^2 + mgy \qquad (3.1.7)$$

Explanation 2:

If the force which the cart receives[11] is given as $F = -kx$ in formula (3.1.5) and the integral calculus is done to both sides of this formula, the result is as follows.

$$F = -kx = -\frac{dV}{dx}$$

$$k\int x\,dx = \int dV$$

$$V = \frac{1}{2}kx^2$$

[10] If the outer way from the center is on (+) direction, the gravity is on (−) direction.

[11] If it receives force of $F = -ma$, which is expressed with the acceleration $a$ , it can be written as below.

$$F = -ma = -\frac{dV}{dx}, \quad ma\int dx = \int dV$$

$$V = max$$

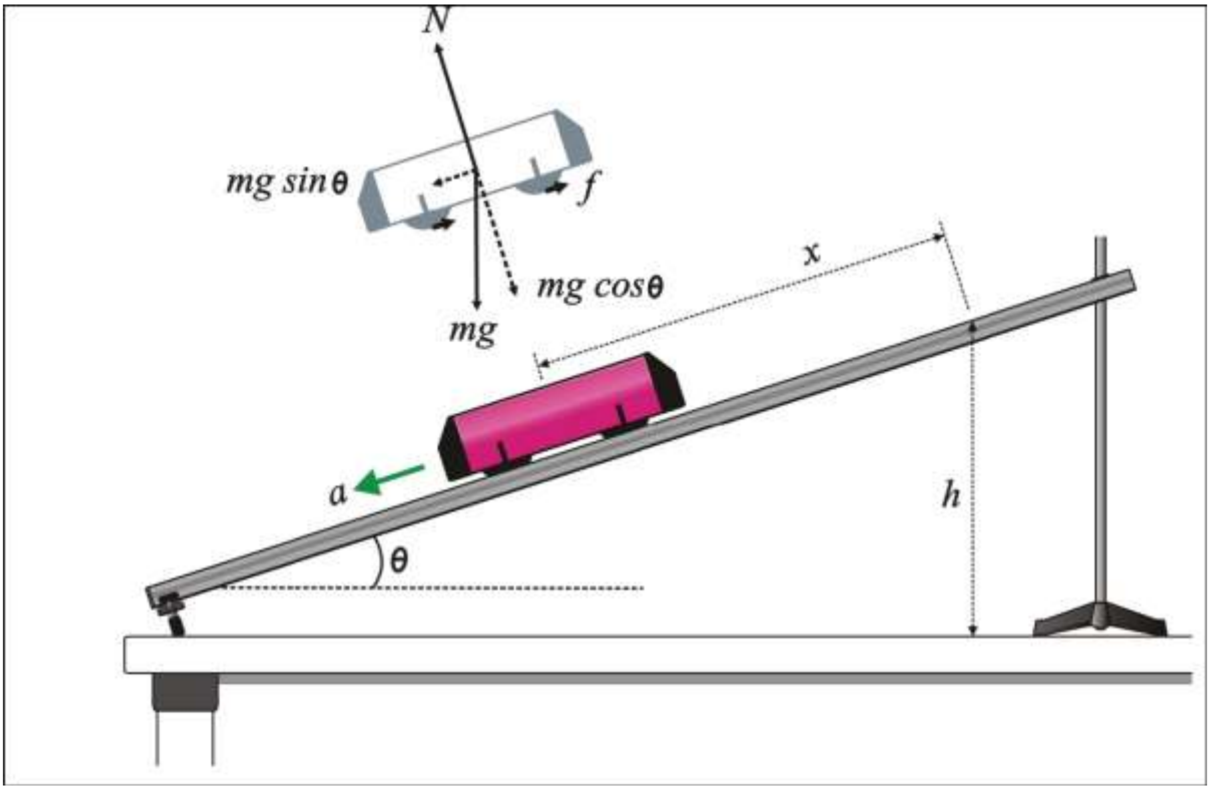
### Exercise 3.1.2: Object's Motion When Constant Force Is Working

1. As in picture 3.1.2, describe the relationship between the cart's location, velocity and mechanical energy on the slide.

2. As in picture 3.1.3, set up the equation of motion for the cart's motion by the pendulum and calculate the cart's acceleration.

Explanation 1:

As in picture 3.1.2's vector diagram, the gravity $mg$ is working on the cart on the slide. Considering the frictional force $f$ [12], the force that moves the cart and the acceleration is as below.




Picture 3.1.2 cart's motion on a slide

[12] The cart's wheels are designed structurally to cause the smallest friction with the track. The frictional force can be ignored according to the situations.

$$F = ma = mg(\sin\theta - \mu\cos\theta)$$
$$a = \frac{F}{m} = g(\sin\theta - \mu\cos\theta) \qquad (3.1.8)$$

Without considering the friction between the track and the cart, calculate from formula 3.1.3 and 3.1.4 the cart's location and velocity starting from the stop state. If the velocity when the cart is coming down on the slide is $v$, the velocity just after it comes down the slide is $v'$ and the slide's length is $L$, then the height $h = L\sin\theta$.

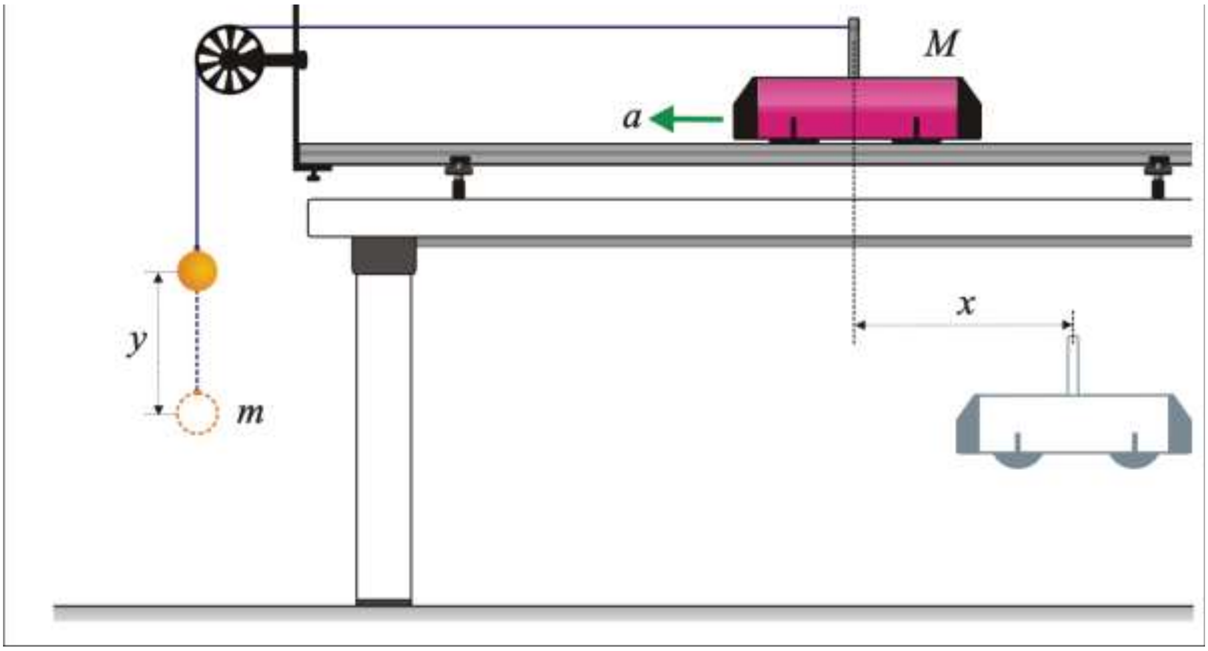
$$x = v_0 t + \frac{1}{2}at^2 = \frac{1}{2}g\sin\theta t^2$$
$$v = \sqrt{v_0^2 + 2ax} = \sqrt{2gx\sin\theta} \qquad (3.1.9)$$
$$v' = \sqrt{2gL\sin\theta} = \sqrt{2gh}$$

The cart's mechanical energy is conserved, so the relationship between the cart's kinetic energy and potential energy is like below.

$$E = \frac{1}{2}mv_0^2 + mgLsin\theta = \frac{1}{2}mv^2 + mg(L-x)\sin\theta$$
$$= mgh = \frac{1}{2}mv'^2 \qquad (3.1.10)$$

Explanation 2:

In picture 3.1.3, when the pendulum of mass $m$ comes down as much as $y$, the cart moves as much as $x$. When the rope's tension is $T$, the force that works on the pulley is $F_r$, the inertia moment is $I$, and the pulley's mass is $M_r$, the equation of motion for the pendulum and the cart is as follows.




Picture 3.1.3 cart's motion by pendulum

$$m\ddot{y} = mg - T - F_r$$
$$M\ddot{x} = T - \mu Mg \qquad (3.1.11)$$

Then $\ddot{x} = \ddot{y}$ $F_r = \frac{I}{r^2}\ddot{x}$[13], so if $T$ is eliminated form two formulae above and the cart's acceleration $\ddot{x}$ is calculated, the result is as follows.

$$m\ddot{y} + M\ddot{x} + \mu Mg + \frac{I}{r^2}\ddot{x} - mg = 0$$
$$\ddot{x} = \ddot{y} = \frac{(m - \mu M)g}{M + m + \frac{I}{r^2}} \qquad (3.1.12)$$

Experimentally, although the resistance between the cart's wheels and track is extremely small, the damping of motion by friction occurs and this results in loss of energy so the graph of mechanical energy conservation shows definite differences[14]. Also the cart's modulus of

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[13] In case of $r = rF_r, \ddot{y} = r\dot{\omega}$, $F_r = \frac{I\dot{\omega}}{r} = \frac{I}{r}\left(\frac{\ddot{y}}{r}\right) = \frac{I}{r^2}\ddot{y}$.

[14] In general curriculum, it is possible not to calculate the modulus of friction $\mu$ but to analyze within the experiment error range when calculating the velocity and acceleration, but in deepened physics curriculum, the friction have to be considered.

friction $\mu$ can be calculated experimentally in picture 3.1.3. In the formula above, the inertia moment of the pulley should be calculated as $I = 1/2 M_r r^2$. If the inertia moment is so small that it is not considered to affect the system, $I = 0$ and $\ddot{x}$ and $T$ in formula (3.1.12) can be rewritten as below.

$$m\ddot{x} = \frac{(m - \mu M)g}{M + m}$$
$$T = \frac{mM(1 + \mu)g}{M + m} \quad (3.1.13)$$

The relationship of mechanical energy conservation according to time can be described as follows with calculating the energy loss by the frictional resistance.

$$T(t) = \frac{1}{2}(M + m)v^2(t) = \frac{1}{2}(M + m)\left[v_0 + v'(t)\right]^2$$
$$V(t) = mg\left[h - y(t)\right] \quad (3.1.14)$$
$$E(t) = T(t) + V(t) + \frac{1}{2}I\omega^2(t) + \mu Mgx(t)$$

When the system's initial velocity is $v_0 = 0$ and the frictional resistance and inertia moment are not calculated, formula (3.1.14) can be simplified as below. The mechanical energy relationship of the cart and the pendulum is like below.

$$mgh = \frac{1}{2}(M + m)v^2 \quad (3.1.15)$$

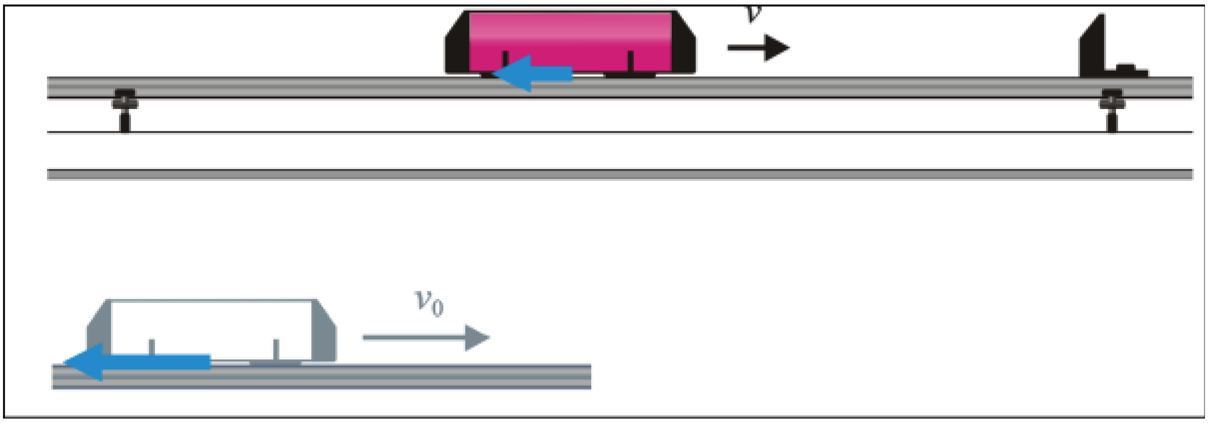
In the experiment of first dimensional track and a cart dealing with velocity and acceleration without calculating the energy, the inertia moment $I$ and the frictional modulus $\mu$ that have small values[15] can be ignored and it is possible to explain and prove Newton's laws more easily within the experiment error range.

[15] The first dimensional track and the cart are consisted to minimize the frictional resistance, and the inertia moment of the pulley is also designed structurally to have very small value.

## Exercise 3.1.3: Object's Motion When Force in Proportion to Velocity Is Working

1. As in picture 3.1.4, calculate the cart's velocity and energy which gets force in proportion to the velocity on the horizontal plane.

<u>Explanation:</u>




Picture 3.1.4 cart's motion which gets resistance in proportion to velocity

Getting the force which is in proportion to the velocity on the horizontal plane, the equation of motion is as below.

$$m\ddot{x} + b\dot{x} = 0$$

Since $mx = -bv$, if this is solved about the velocity $v$, the result is like below.

$$-\frac{m}{b}\int_{v_0}^{v}\frac{1}{v}dv = \int_{t_0=0}^{t}dt, \quad \ln\left[\frac{v}{v_0}\right] = -\left(\frac{b}{m}\right)t$$

$$\therefore v = v_0 e^{-\frac{b}{m}t} \qquad (3.1.16)$$

The cart's kinetic energy is $T = 1/2\, m\dot{x}^2$ and the rate of energy loss dE/dt is as formula (3.1.17)

$$dT/dt = (m\ddot{x})\dot{x} = (-b\dot{x})\dot{x} = -b\dot{x}^2 \qquad (3.1.17)$$

### 3.1.2 Lagrangian Mechanics

Until now, we have explained an object's motion with Newton's equation of motion. From now on, we will do the same process with Lagrangian equation of motion[16].

When the mass $m$ is moving within the potential well[17] $(V(x,t)$ and the system's resistance is small, Lagrangian can be expressed as below.

$$L(x_i,\dot{x}_i,t) = T - V = \frac{1}{2}m\dot{x}^2 - V(x,t) \qquad (3.1.18)$$

And the general form of Lagrangian equation expressed with the generalized coordinate and velocity is like below.

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{x}} - \frac{\partial L}{\partial x} = 0 \qquad (3.1.19)$$

With formula (3.1.19) the object's equation of motion can be solved. When using Lagrangian in a system that conserves the momentum and energy, follow the process below.

a. Choose a coordinates system that can describe the object's motion well.

b. Calculate the kinetic energy $T$ with the value of time differential, and the potential energy $V$ with the value of location.

c. Get the equation of motion using formula (3.1.19).

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[16] If you use Lagrangian, you can calculate physical situations that the object is restricted complicatedly by the motion orbit very easily with scalar energy function without calculating vector physical values such as velocity, momentum and force.

[17] Potential is the potential energy that is expressed by the object's function of location only within the place in which the conservative power is working and the power is limited only to the object's location. Potential wall means the spatial range that can express the object's location with potential.

### Exercise 3.1.2: Equation of Motion of an Object that Is Free Falling

Calculate the motion of an object that is moving with gravity $F = -mg$ .

Explanation:

If formula (3.1.18) is applied to formula (3.1.19), the result is as follows.

$$\frac{\partial L}{\partial y} = \frac{\partial (T-V)}{\partial y} = -\frac{\partial}{\partial y}(mgy) = -mg$$

$$\frac{\partial L}{\partial \dot{y}} = \frac{\partial (T-V)}{\partial \dot{y}} = \frac{\partial}{\partial \dot{y}}\left(\frac{1}{2}m\dot{y}^2\right) = m\dot{y}$$

So you can get the equation of motion below.

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{y}} - \frac{\partial L}{\partial y} = \frac{d}{dt}(m\dot{y}) - mg = 0$$

$$\therefore m\ddot{y} + mg = 0 \qquad (3.1.20)$$

Formula (3.1.20) shows that it becomes Newton's equation of motion. This indicates that potential is not a function of time but the most general equation of motion of the first dimensional motion.

### Exercise 3.1.3: Equation of Motion of an Object that Gets Constant Force

In picture 3.1.1, solve the equation of motion of a cart that is moving with elastic force $F --mg$ when the friction between the track and the cart is ignored.

Explanation:

Solve this equation with Lagrangian function just as in exercise 3.1.2. If the potential energy by the elastic force $V=\frac{1}{2}kx^2$ V is substituted, the result is as follows.

$$\frac{\partial L}{\partial x}=\frac{\partial(T-V)}{\partial x}=-\frac{\partial}{\partial x}\left(\frac{1}{2}kx^2\right)=-kx$$

$$\frac{\partial L}{\partial \dot{x}}=\frac{\partial (T-V)}{\partial \dot{x}}=\frac{\partial}{\partial \dot{x}}\left(\frac{1}{2}m\dot{x}^2\right)=m\dot{x}$$

So you can get the equation of motion as below.

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{x}}-\frac{\partial L}{\partial x}=\frac{d}{dt}(m\dot{x})-kx=0$$

$$\therefore m\ddot{x}+kx=0 \qquad (3.1.21)$$

This equation of motion is applied only until the cart gets the spring's force and after escaping from the spring, the cart is finally in the state of uniform motion. Concerning formula (3.1.21), chapter 5.Oscillation deals with and explains the system's motion that gets the spring's force with simulations and analysis process. As in formula (3.1.19) it is possible to set up an object's equation of motion as physics model, to find solution based on modeling[18] and to analyze data.

[18] Refer chapter 2.Physics Modeling-based Data Analysis. Physics model can be helpful for learning from both theory and experiment so practice chapter 2 enough along with the finding solution questions.

# 3.2

## Simulation

This is a process of understanding how the motion's graph changes according to the acceleration in the simulation of force and motion[19]. By changing acceleration in various physical situations, it will be possible to understand the concepts of velocity and acceleration with the graph. With simulations, physical situations that are not easy to experiment or explain by theories can be understood, so I hope this will solve students' difficulties. Simulations can be omitted according to the educational situations.

To draw an object's location and velocity graph by simulations in Excel, you should use Newton's equations of motion such as formula (3.1.3) and (3.1.4) explained before and Excel's VBA(visual basic for applications).[20]

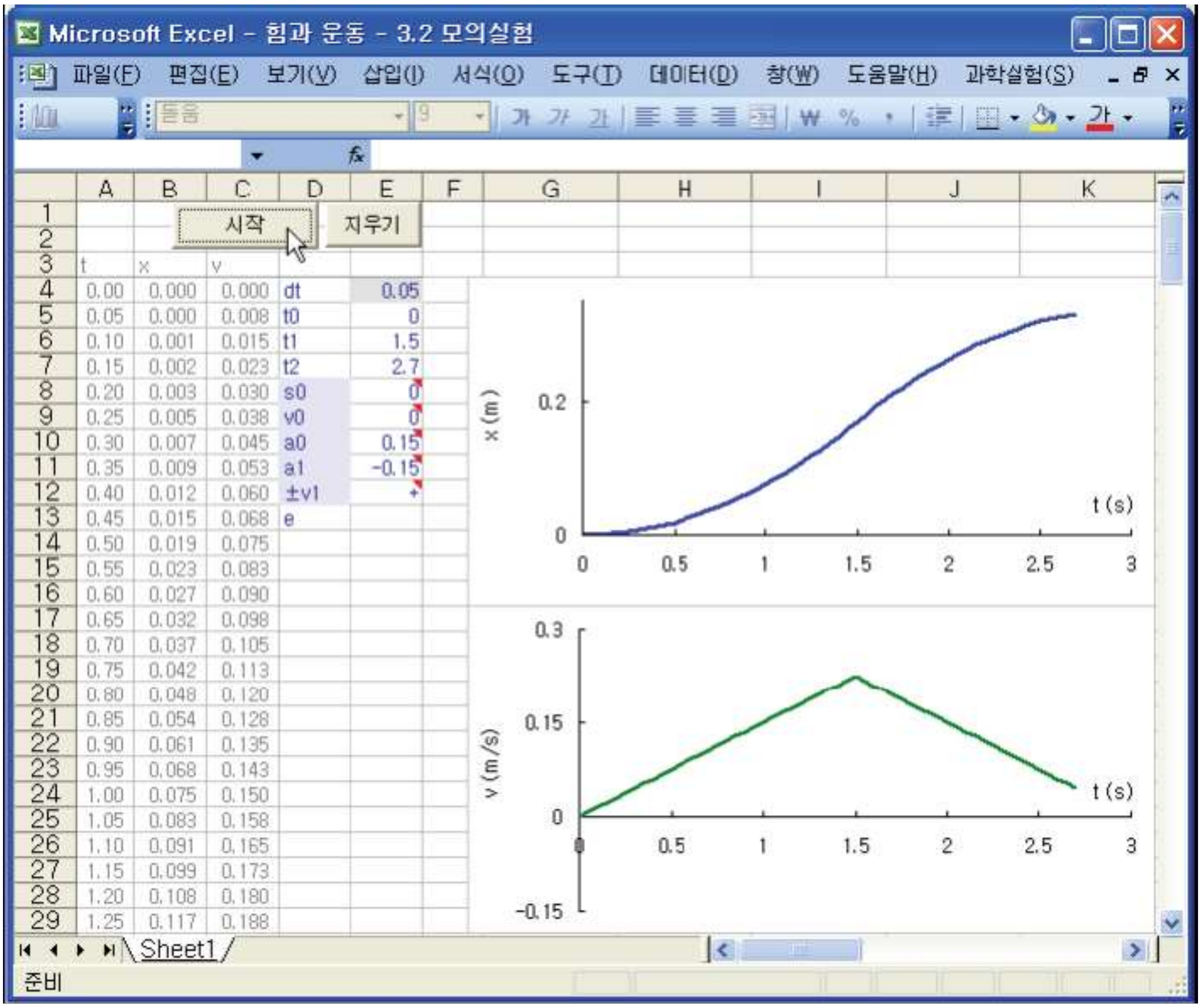
[19] This simulation can be used not only in AP(advanced placement) level but also in low grade level. With this, I hope that general difficulties which the students have in force and motion such as GAP(graph as picture) and SHC(slope-height confusion) may be solved.

[20] Excel VBA can be drawn as the original codes in VBE(visual basic editor), and the VBA original codes for simulations can be downloaded at www.sciencecube.com

### 3.2.1 Simulation Making

The process of simulation making can usually be omitted in the educational process. However, with this process, you can find out control factors and initial conditions that have to be considered in the real experimental situations. This simulation making process also can be the base of explaining various physical situations theoretically. Also, it will be helpful to improve the ability of making computer programs needed in the physics experiment analysis process of high level.

Picture 3.2.1 is a graph of distance and velocity drawn by simulating a cart's motion on the first dimensional motion track. You should record the results value of time, location and velocity in row A, B, and C, and the initial values from E4 to E13.




Picture 3.2.1 simulation of distant and velocity graph: the simulation can be operated by clicking [Start] button in worksheet.

As in picture 3.2.1, the simulation is operated by clicking [Start] button in "Sheet 1", so you can compare and interpret $x-t$, $x^{\cdot}-t$ graphs calculated by inputting initial conditions variously.

If the factors are designed as the initial conditions, the result is as below.

| | |
|---|---|
| dt | time(s) interval in graph |
| t0 | the first time |
| t1 | the second time |
| s0 | the first location |
| v0 | the first velocity |
| a0 | the first acceleration |
| a1 | the second acceleration |
| ±v1 | direction at the velocity of time t1[21] |
| e | loss rate of velocity[22] v1 at time t1 |

Table 3.2.1 initial condition factors of simulation

The simulation in picture 3.2.1 is the case that an objects which started moving with initial location s0, velocity v0, and acceleration a0 at time t0 moves with acceleration a1 from time t1 to time t2. The velocity v1 at time t1 is calculated by the first acceleration a0. This simulation can be substituted to the cart's motion on the first dimensional track.

First, if the object's location s1 and velocity v1 between time t0 and time t1 are calculated, the result is as follows.

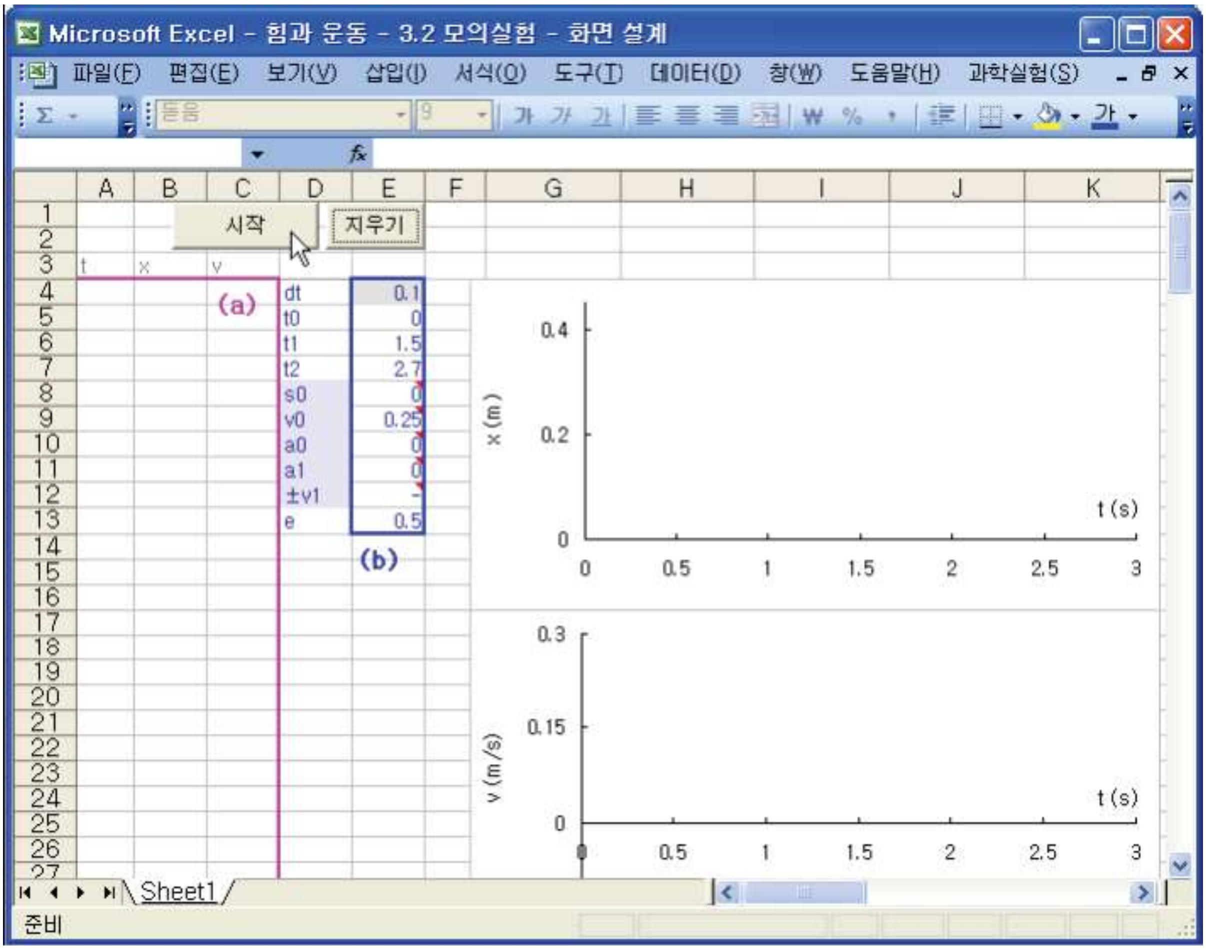
$$s_1 = s_0 + v_0 t + \frac{1}{2} a_0 t^2 \qquad (3.2.1)$$
$$v_1 = v_0 + a_0 t$$

Second, if the object's location s1 and velocity v1 between time t1 and time t2 are calculated, the result is as follows.

[21] When an object that moves with the acceleration a0 from time t0 to time t1 changes the direction at time t1, input the sign "−".
[22] This is the rate of conserving velocity v1 at time t1.

$$s_2 = s_1 \pm v_1(t-t_1) + \frac{1}{2}a_1(t-t_1)^2$$
$$v_2 = \pm e \cdot v_1 + a_1(t-t_1) \qquad (3.2.2)$$

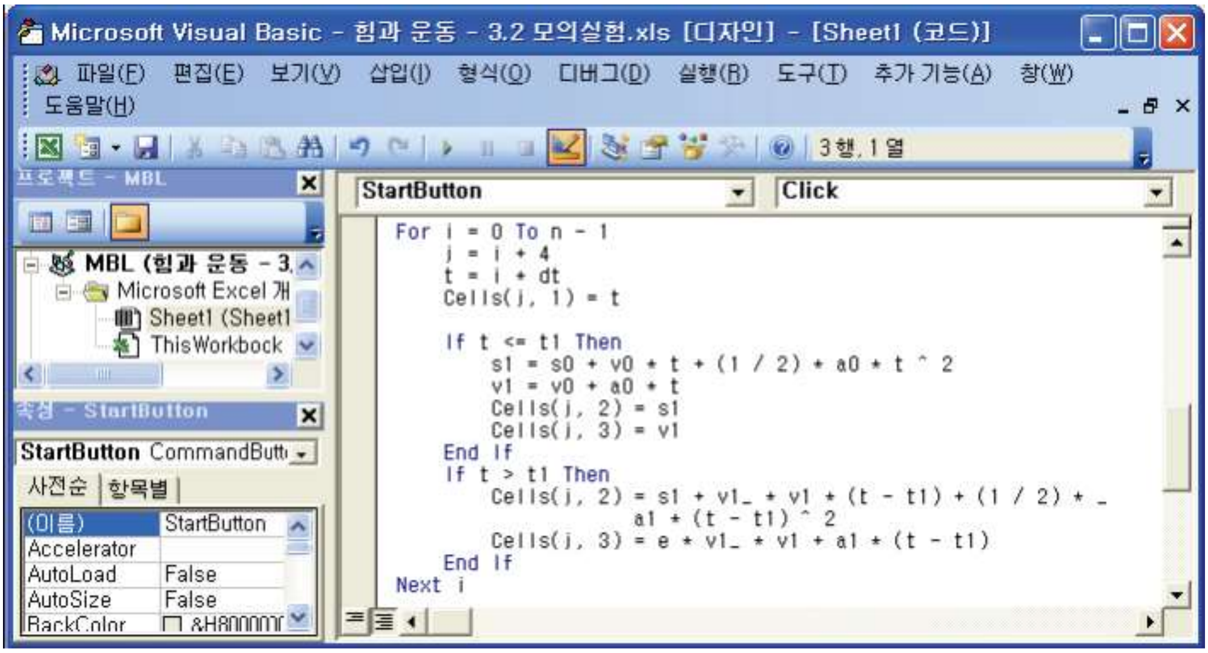
To calculate the object's location $x$ and velocity $v$ with the simulation, the formulae that interpret Newton's equation of motion mathematically should be applied in Excel VBA codes. Picture 3.2.2 is a design planned previously to make simulations in Excel worksheet. In cell A, B, and C included in (a), the values of time t, location x and velocity v are recorded to each cells successively every certain time interval dt. (b) is the area to input the initial factors of table 3.2.1.




Picture 3.2.2 scene design of simulation

After designing the scene, make VBA code with formula (3.2.1) and (3.2.2). Picture 3.2.3

is a part of the original code made like this. After completing VBA code like this[23], you can start simulation by clicking [Start] button.




Picture 3.2.3 part of VBA code making[24] scene for simulation

### 3.2.2 Operating Simulation

With the simulation, draw charts of distance-time and of velocity-time by changing the initial conditions. With exercises, the simulations for various physical situations will be dealt with. The initial values of simulations will be from the supposition of a cart's motion on the first dimensional track which will be used in the real experiment. Also, with the simulations, predict the real experiment situations within the experiment error range and explain them.

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[23] The VBA code is saved when saving Excel workbook, so you can open the workbook again next time and use it.

[24] Refer to the supplement of this book for the VBA original code download site.

Exercise 3.2.1: Cart's Motion on a Slope

As in the experiment situation of picture 3.1.2, when the slope is 0.8m, and the angle of the slope is $3.0 \pm 0.2^\circ$, draw the graph of the cart(with mass 0.525kg)'s moving distance and velocity, and learn about the conservation of mechanical energy.

Explanation:

Not considering the friction, the cart's acceleration on the slope is $a = g \sin\theta$, so $a = 9.8 \times 0.052 = 0.513$. The initial conditions for this situation are as table 3.2.2.

| | |
|---|---|
| dt | 0.1 |
| t0 | 0 |
| t1 | 1.7 |
| t2 | 1.7 |
| s0 | 0 |
| v0 | 0 |
| a0 | 0.513 |
| a1 | 0 |
| ±v1 | |
| e | |

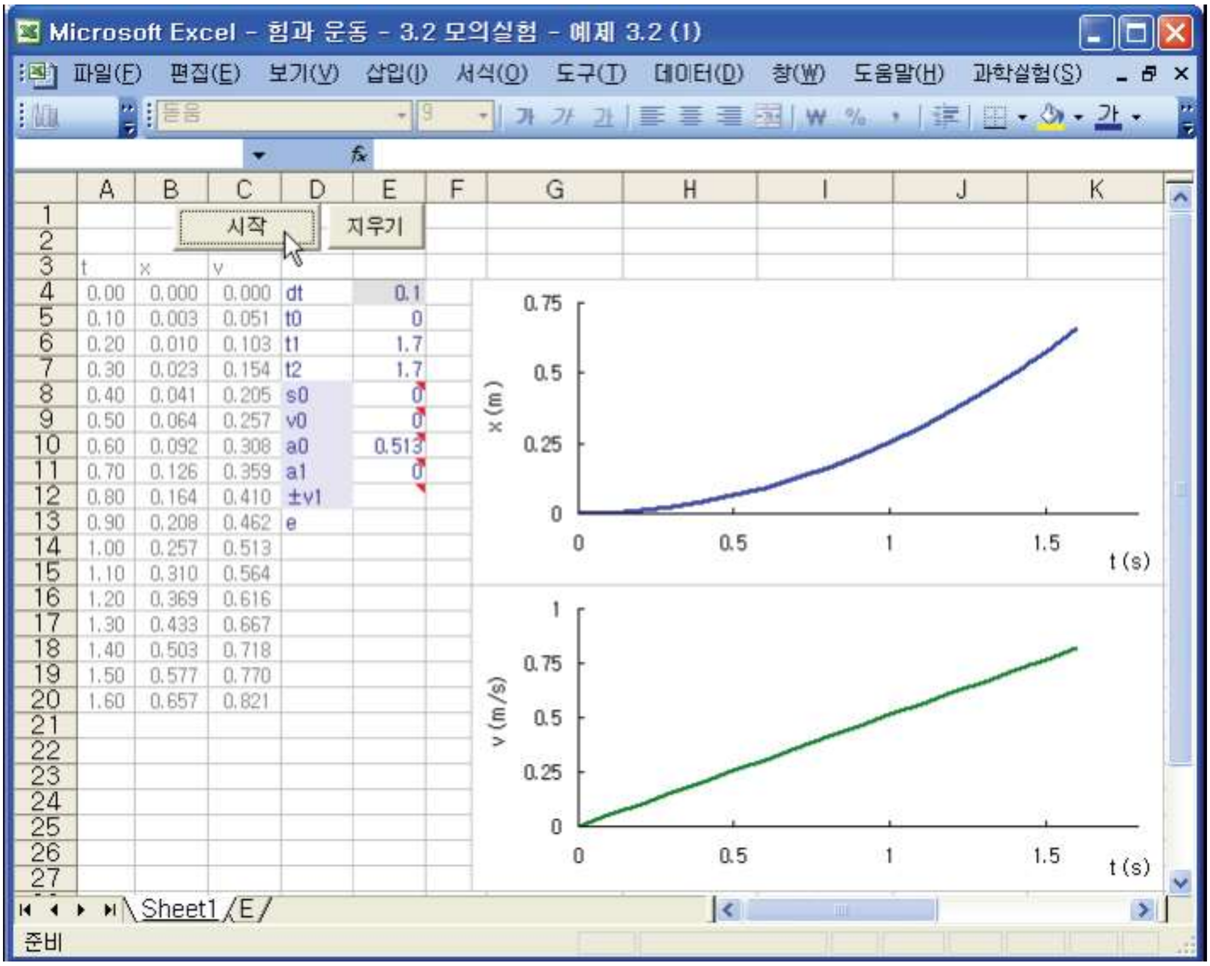
Table 3.2.2 initial conditions used in the simulation of a cart's motion on a slope

If the graph of location and velocity is drawn by inputting values of table 3.2.2, it is the same as picture 3.2.4. By calculating the cart's kinetic energy T and potential energy V as below, the mechanical energy E can be calculated. If the graph for the conservation of mechanical energy is drawn with these values, the result is as follows.

$$T=\frac{1}{2}mv^2=(0.5\times0.525)\cdot v^2$$

$$V=mgh=mg\sin\theta(L-x)$$

$$=(0.525\times9.8\times0.052)\cdot(0.8-x)$$



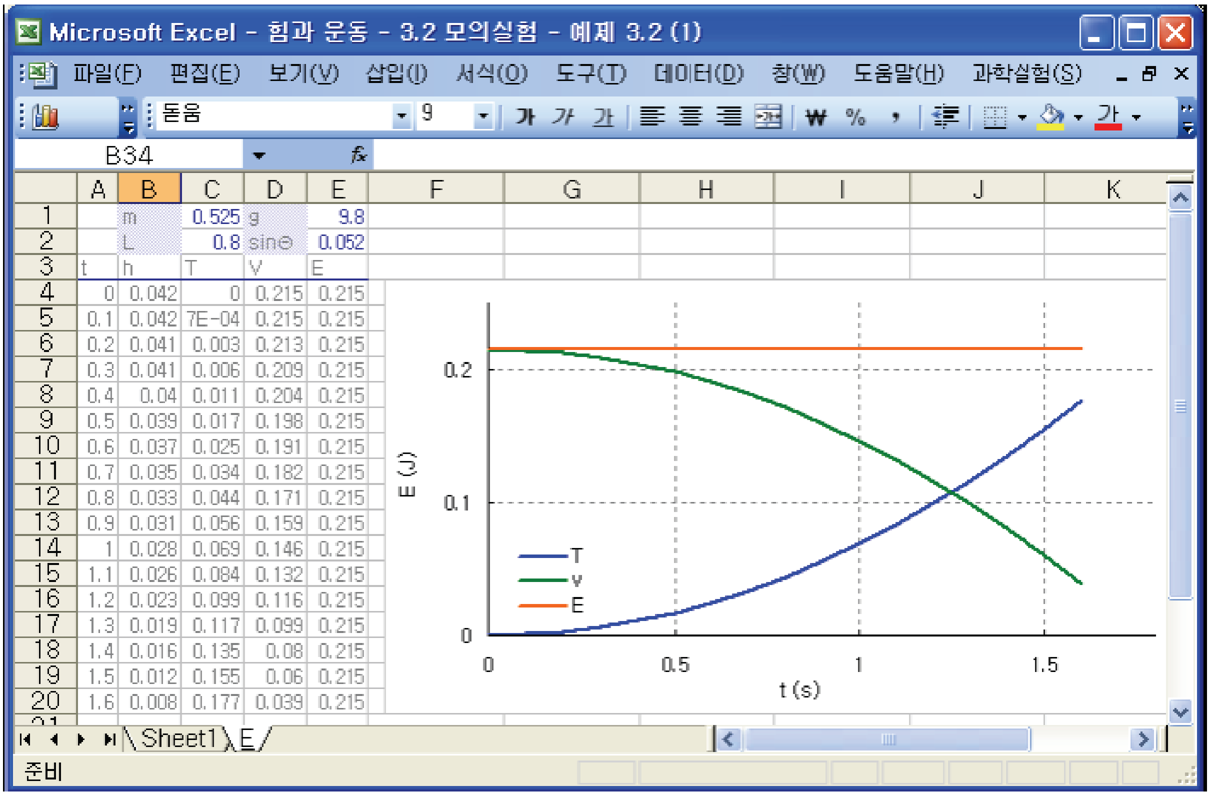

Picture 3.2.4 $x-t$ and $v-t$ graph for a cart's motion on a slope

After operating the simulation in "Sheet1" of Excel workbook, make a new worksheet "E". Sheet E is to draw the graph for the conservation of mechanical energy. In sheet E, record the cart's mass m in cell C1, the slope's length L in C2, the value of $\sin\theta$ in E2, and input formula as table 3.2.3 so that it can calculate the kinetic energy T in row C, potential energy in row D, and total energy E[25] in row E.

[25] In the real experiment, there will be differences in the graph because of the energy loss caused by friction.

| | |
|---|---|
| row B (h) | =+ ($C$2-Sheet1!B4)*$E$2 |
| row C (V) | =1/2*($C$1*Sheet1!C4^2) |
| row D (T) | =+ $C$1*$E$1*B4 |
| row E (E) | =+ C4+ D4 |

Table 3.2.3 inputting formula concerning energy V, T and E in worksheet




Picture 3.2.5 graph of energy E, T and V concerning a cart's motion on a slope[26]

### Exercise 3.2.2: Cart's Motion that Changes Its Direction

In picture 3.2.6, a cart moves with constant velocity $v_1 = 0.244$, and then at the right end, it is opposed by the magnet bumper ,changes the direction and moves with velocity $v_2$. From the rate of velocity e=0.997 and time $t_1$=0.8 before and after the opposition to after the direction is changed $t_2$=1.6, the $x-t$ and $v-t$ graph of this cart is as picture 3.2.7.

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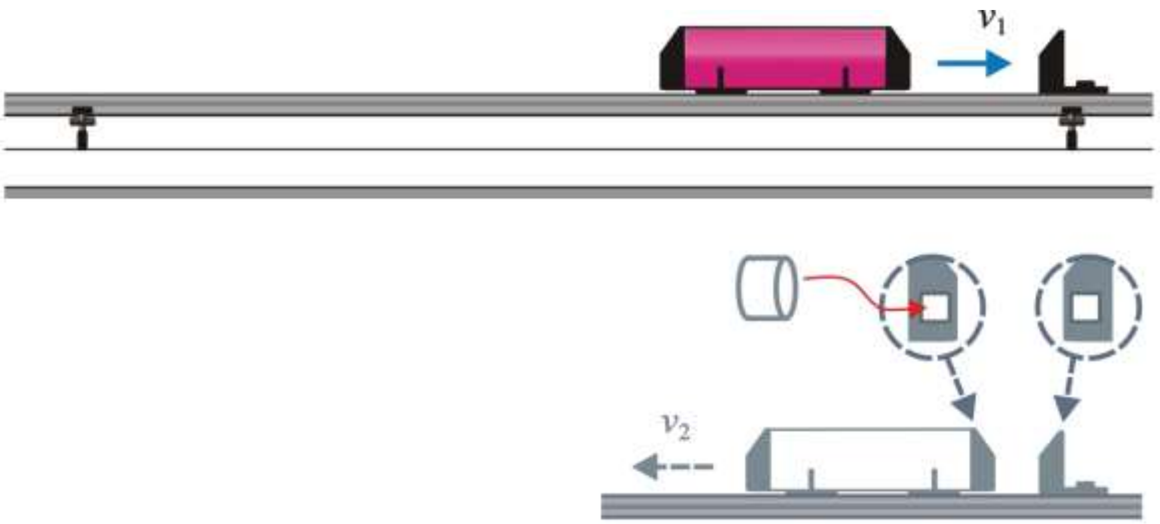
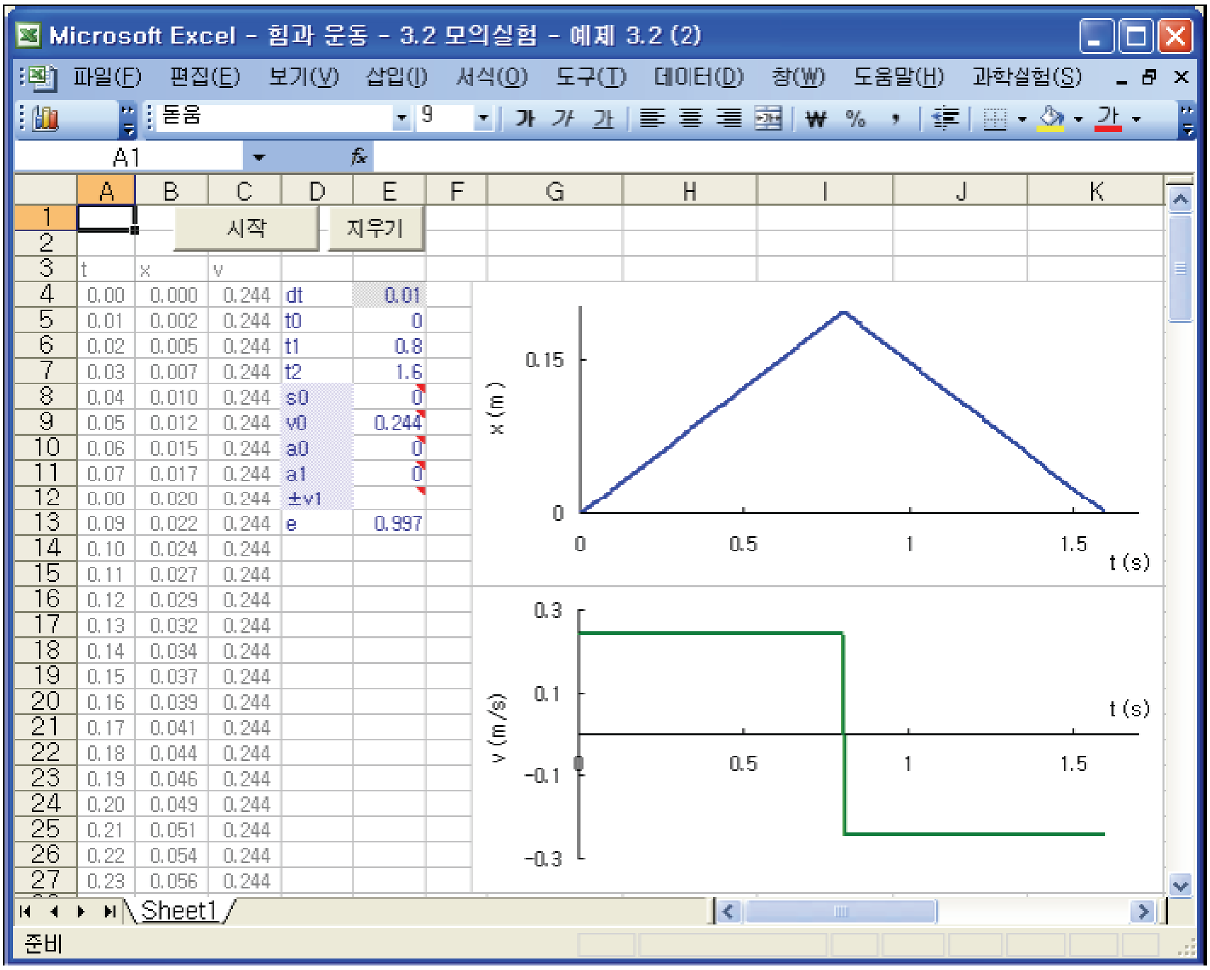
[26] Experimentally, by considering the energy loss R caused by friction, draw graph of E=T+ V+ R.




Picture 3.2.6motion in which the cart's direction is changed because of the opposition caused by magnet bumper[27]




| | A | B | C | D | E |
|---|---|---|---|---|---|
| 3 | t | x | v | | |
| 4 | 0.00 | 0.000 | 0.244 | dt | 0.01 |
| 5 | 0.01 | 0.002 | 0.244 | t0 | 0 |
| 6 | 0.02 | 0.005 | 0.244 | t1 | 0.8 |
| 7 | 0.03 | 0.007 | 0.244 | t2 | 1.6 |
| 8 | 0.04 | 0.010 | 0.244 | s0 | 0 |
| 9 | 0.05 | 0.012 | 0.244 | v0 | 0.244 |
| 10 | 0.06 | 0.015 | 0.244 | a0 | 0 |
| 11 | 0.07 | 0.017 | 0.244 | a1 | 0 |
| 12 | 0.00 | 0.020 | 0.244 | ±v1 | |
| 13 | 0.09 | 0.022 | 0.244 | e | 0.997 |
| 14 | 0.10 | 0.024 | 0.244 | | |
| 15 | 0.11 | 0.027 | 0.244 | | |
| 16 | 0.12 | 0.029 | 0.244 | | |
| 17 | 0.13 | 0.032 | 0.244 | | |
| 18 | 0.14 | 0.034 | 0.244 | | |
| 19 | 0.15 | 0.037 | 0.244 | | |
| 20 | 0.16 | 0.039 | 0.244 | | |
| 21 | 0.17 | 0.041 | 0.244 | | |
| 22 | 0.18 | 0.044 | 0.244 | | |
| 23 | 0.19 | 0.046 | 0.244 | | |
| 24 | 0.20 | 0.049 | 0.244 | | |
| 25 | 0.21 | 0.051 | 0.244 | | |
| 26 | 0.22 | 0.054 | 0.244 | | |
| 27 | 0.23 | 0.056 | 0.244 | | |

Picture 3.2.7 $x-t$ and $v-t$ graph of a cart's motion that changes its direction

27 Inside the cart's bumper and the stop bumper, round neodymium magnets can be put. If the same poles face to each other, the cart's momentum is nearly preserved (e≈0.95~0.98) and it is opposed to change the direction.

# 3.3

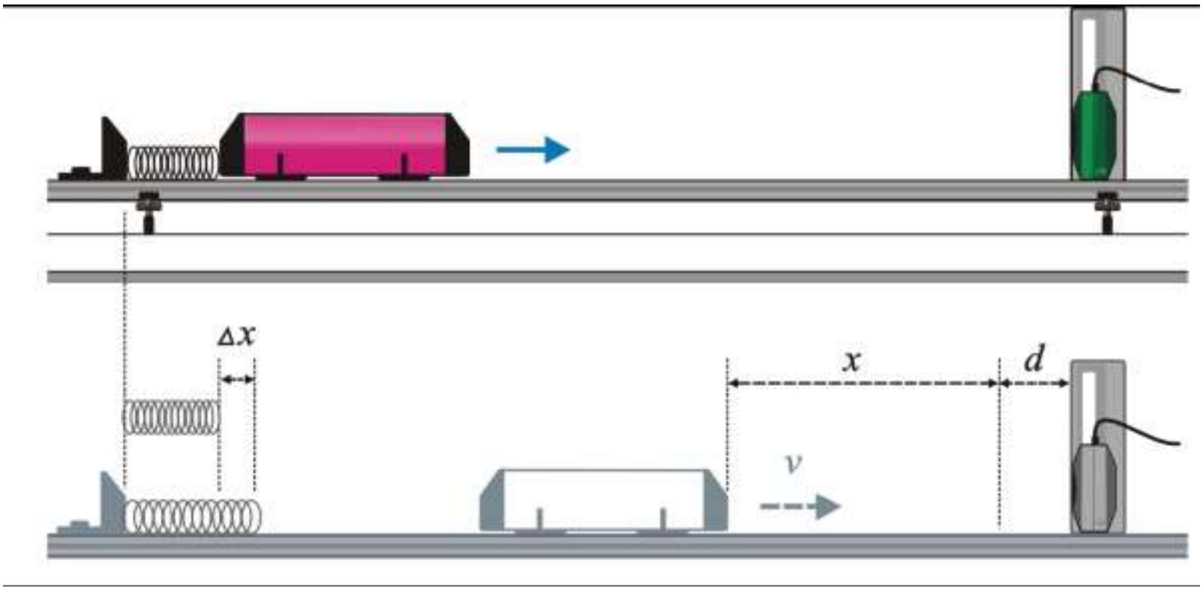
# Experiment Analysis

Experiment Analysis is the process to mathematically calculate and analyze data collected no by simulations but by real physics experiment. This process includes measuring a cart's location on the first dimensional track by motion sensor and defining and analyzing formulae for velocity, momentum and energy in Excel.

If the cart's location $x$ is measured thorough the experiment, it can be used by making VBA program of Excel workbook which calculates the velocity $v$ and the acceleration $a$. Therefore, the momentum concerning a system's motion can be calculated and the motion can be explained. Also, formulae to analyze the experiment results in various experiment situations can be written in cells of worksheet and this will be explained with examples. One way to analyze the velocity and acceleration in Excel is to input the formula directly in the cell of worksheet, and the other way is to write and use VBA program[28]. Let's inquire the latter.

[28] It is convenient to use Excel workbook containing VBA program when doing experiments repeatedly. Refer to chapter 5 to understand the making of VBA program.

### 3.3.1 Motion with Regular Direction: Location and Velocity



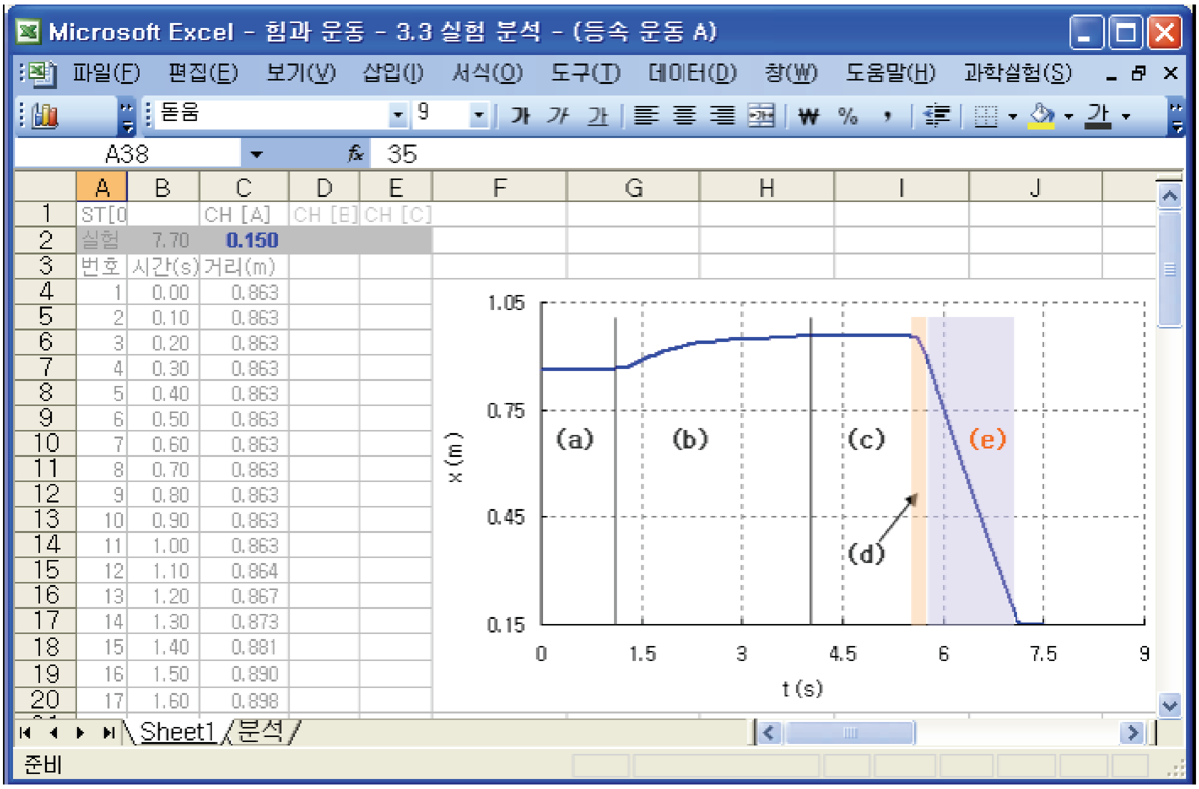

Picture 3.3.1 experiment of a cart's motion: the distance that the cart moves should be measured by the motion sensor. The measured value is the cart's location on the track.

Analyze the cart's velocity with the example of picture 3.3.1. Picture 3.3.1 is the situation that makes the cart move with regular velocity by pushing the cart on the track with a push-pull spring. The cart's location should be measured with the motion sensor. If you compress the spring as much as $\Delta x$ and release the cart, the cart is pushed by the spring's elasticity and the cart will be in uniform motion. To move the cart, you can use not only the spring but also other methods such as elastic strings, fan and so on. According to the methods, you can make the cart's motion as either uniform motion or uniform acceleration motion. Also, you can design the experiment by changing the motion sensor's setting up so that it can sense[29] the cart's location according to the experiment situations.

Picture 3.3.2 is the scene of opening "Sheet 1" of Excel which collected data in the experiment situation above. When you connect the motion sensor with channel A and collect data in "Sheet 1", the time data is recorded in row B, and the cart's location data in row C. Many sheets can be added in Excel workbook, so you can experiment repeatedly in the same experiment situation and save in as one workbook file.

[29] In picture 3.3.1, d(=15cm) is not the distance that the motion sensor can sense.

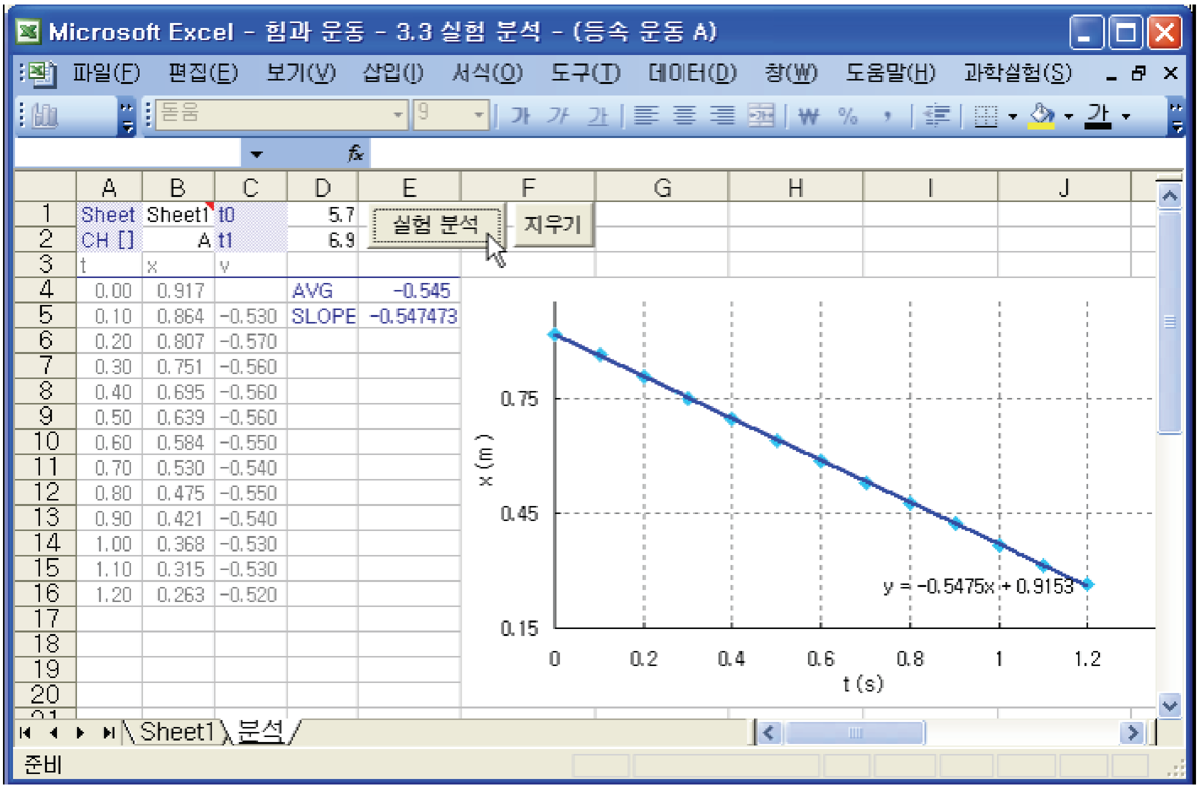
Although the experiment situations are different, the experiment data is collected in worksheets as this way so regardless of the experiment situations and the initial conditions, the way to analyze the velocity of uniform motion can be applied as the same. For example, regardless of using the spring or pushing the cart directly with hands, the process of analyzing the oscillation's experiment data is the same.




Picture 3.3.2 result of collecting data in "Force and Motion (Uniform Motion A).xls" in Excel: (a) the section that the cart is stopped (b) the section that the spring is compressed by moving the cart (c) the section that the spring is compressed and stayed the same (d) the section that the cart is released and pushed by the spring (e) the section that the cart is moving

As in picture 3.3.2, using "sheet1" in which the experiment data of time and location are recorded, the experiment analysis can be done in "Analysis"[30] sheet. The way of writing the analysis sheet for the experiment analysis[31] can improve the ability of experiment analysis for the students of high level physics experiments by executing such process on their own. After collecting data in Sheet 1, start analyzing by opening "Analysis" sheet as in picture 3.3.3.

[30] Use "Force and Motion (Uniform Motion A).xls" file.
[31] Refer to VBA original codes.

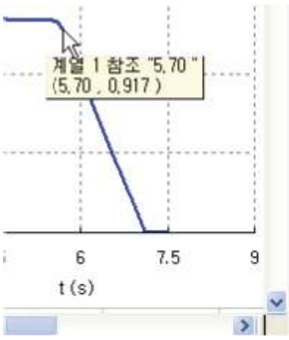



Picture 3.3.3 worksheet that the data of uniform motion experiment is analyzed[32]

In picture 3.3.3, input the analyzing sheet name in cell B1 and the channel connected to the motion sensor in B2. Input the starting time t0 in cell D1 and the finishing time t2 in D2. For example, when the total experiment time is 50 seconds, if the uniform motion interval is from 5.7 to 6.9 second, input 5.7 in t0 and 6.9 in t1. As in picture 3.3.2, the analyzing time interval corresponds to (e) area. If the cursor is on the graph's curve, the values of x axis and Y axis are shown in the balloon message[33], so you can read the value of the time interval that will be analyzed.

---

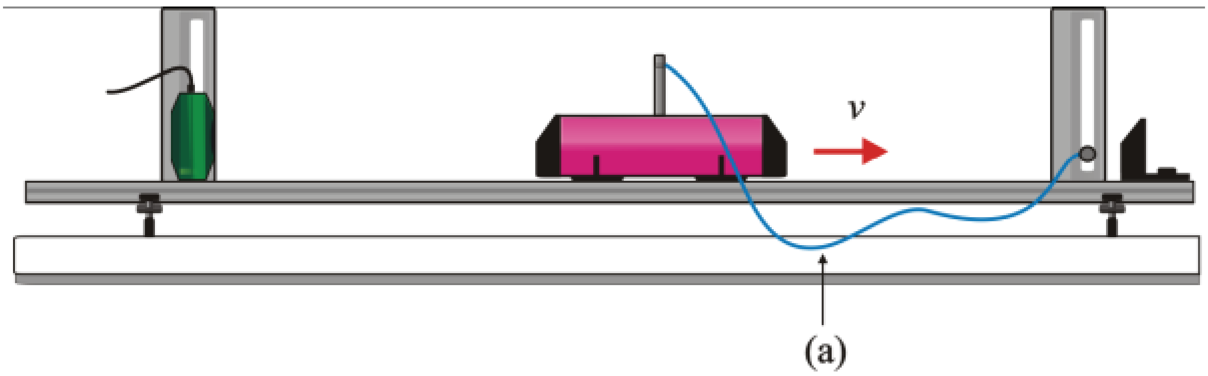
[32] Record the time interval that will be analyzed in cell D1 and D2 and click [Experiment Analysis] button, then the analyzed results will be recorded in cells and the x | t will be drawn in the chart made previously.



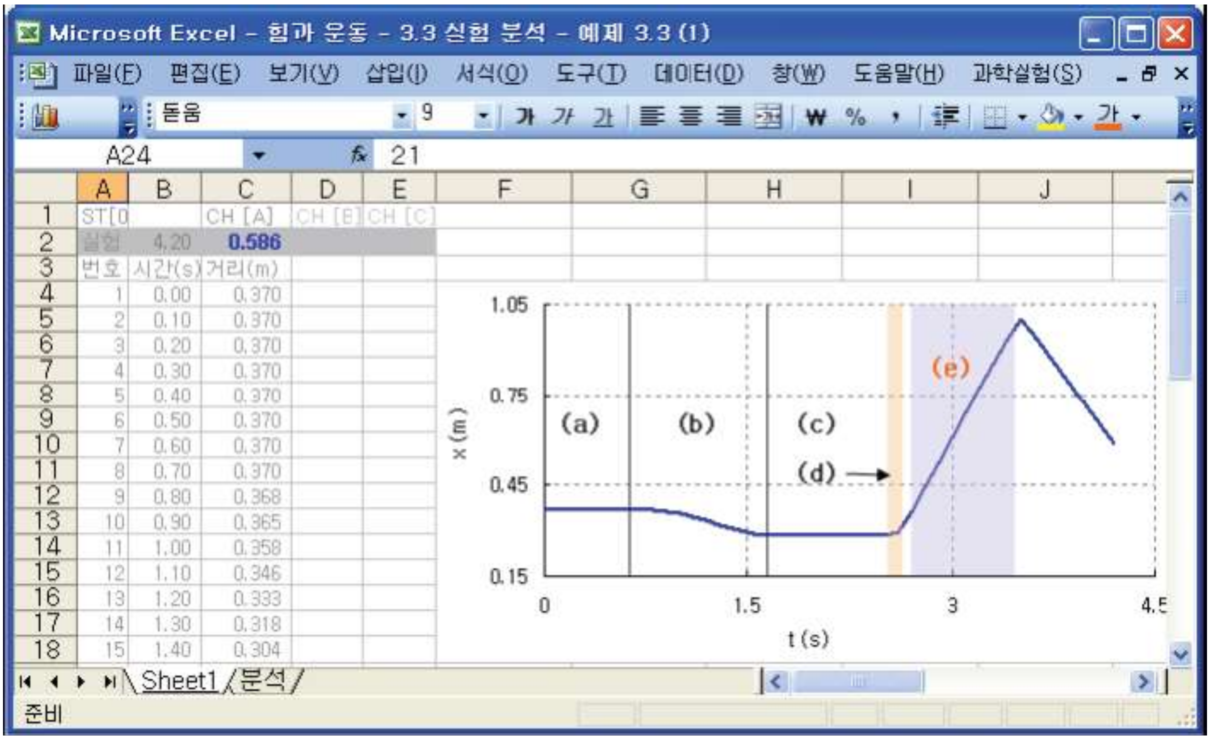

[33] If you put the cursor on the graph's curve, the value of data series (x, y) will be shown in the balloon message.

## Exercise 3.3.1: Cart's Motion Caused by an Elastic Cord

As in picture 3.3.4, calculate the cart's velocity by connecting an elastic cord to it and pulling and releasing it.

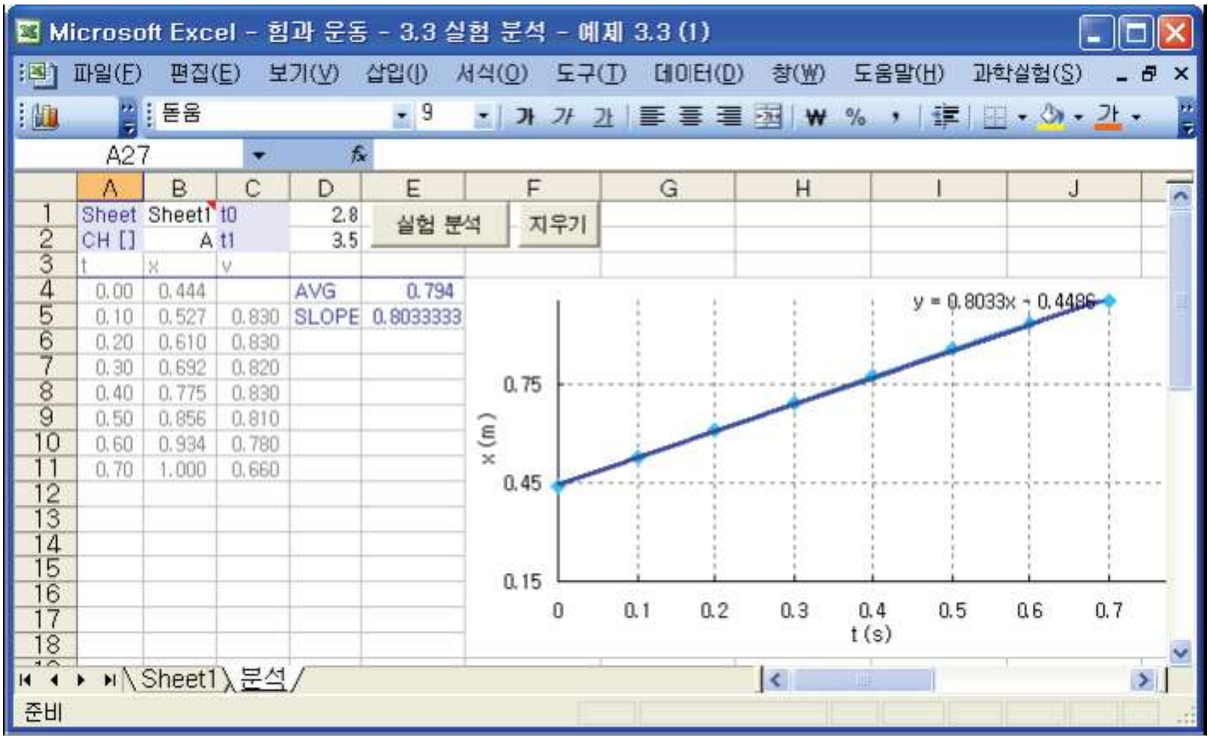



Picture 3.3.4 cart's motion by elastic cord




Picture 3.3.5 result graph of cart's motion by elastic cord

In picture 3.3.5, the data is collected within the section (a), and the cart is pulled left in (b) and it is held in (c) and released in (d), so it finally moves uniformly in (e). You can calculate the cart's velocity by analyzing section (e). The result is as picture 3.3.6, and the velocity can be read as 0.803m/s, which is the value of cell E5. Within the error range, the average value of the velocity 0.794m/s, which is the value of cell E4, can mean the cart's velocity.




Picture 3.3.5 $x-t$ graph of the experiment's result analysis

### Exercise 3.3.2: Cart's Motion Caused by a Spring

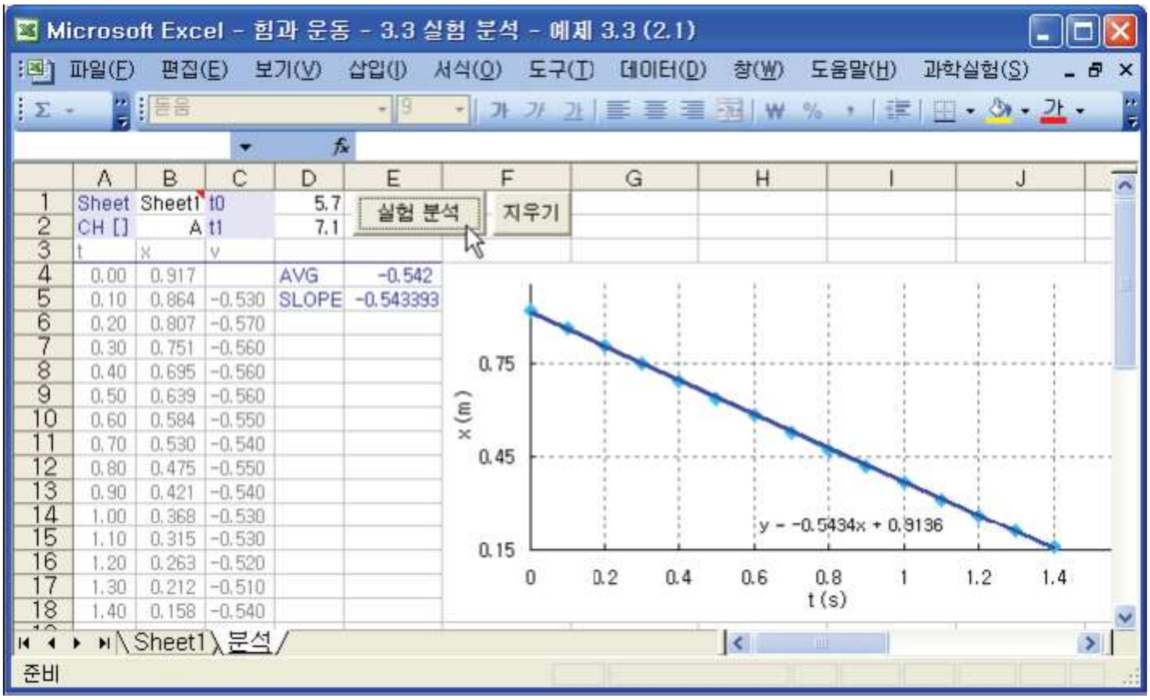
In the situation of picture 3.3.1, compress a spring with the modulus of elasticity 22.895N/m as much as 0.093 m and move the cart.

1. Calculate the velocity $v$ when the cart's mass is 0.525kg and the time that the cart is accelerated by the spring $\Delta t$.

2. Draw the $x-t$ graph by changing the spring's compressed length and calculate the ratio of velocity according to the ratio of length.

Explanation 1:

Picture 3.3.7 is the result of experiment analysis after doing the experiment in "Sheet1" and collecting the data of uniform motion section in "Analysis" sheet.




Picture 3.3.7 experiment result of a cart's motion by a spring

The cart does accelerated motion with the force it gets by the spring during time $\Delta t$, so the acceleration $a$ is as follows.

$$(22.895 \times 0.093) = (0.525) \cdot a$$

$$a = 4.056 \mathrm{m/s^2}$$

And the cart's velocity is $v = -0.544 m/s$, so $\Delta t$ can be calculated as below.

$$v - 0 = a \Delta t$$

$$\therefore \Delta t = 0.544 / 4.056 = 0.134 \mathrm{s}$$

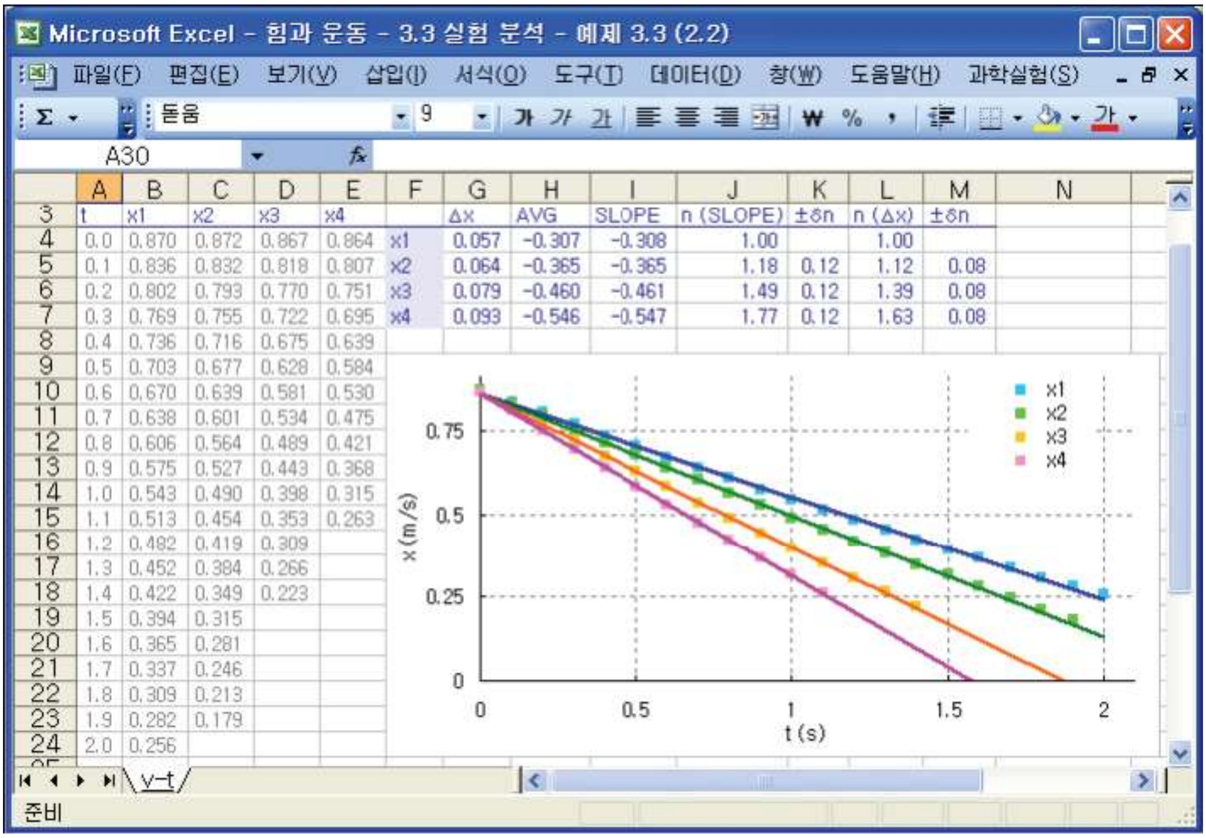
When the cart is accelerated, it gets the force during $\Delta t$, and after it is released form the spring, it does the uniform motion with velocity $v$. Besides calculating $\Delta t$ in this way, if you want to estimate it directly with the experiment, you can make the estimating interval shorter and do the experiment.

Explanation 2:

Picture 3.3.8 is the result of experiment by changing the spring's compressed length. The result of analysis which calculates the ratio of velocity according to the ratio of the spring's compressed length is as table 3.3.1.

| | Δx | AVG | SLOPE | n (SLOPE) | ±δn (SLOPE) | n (Δx) | ±δn (Δx) |
|---|---|---|---|---|---|---|---|
| x1 | 0.057 | -0.307 | -0.308 | 1 | | 1 | |
| x2 | 0.064 | -0.365 | -0.365 | 1.18 | 0.12 | 1.12 | 0.08 |
| x3 | 0.079 | -0.460 | -0.461 | 1.49 | 0.12 | 1.39 | 0.08 |
| x4 | 0.093 | -0.546 | -0.547 | 1.77 | 0.12 | 1.63 | 0.08 |

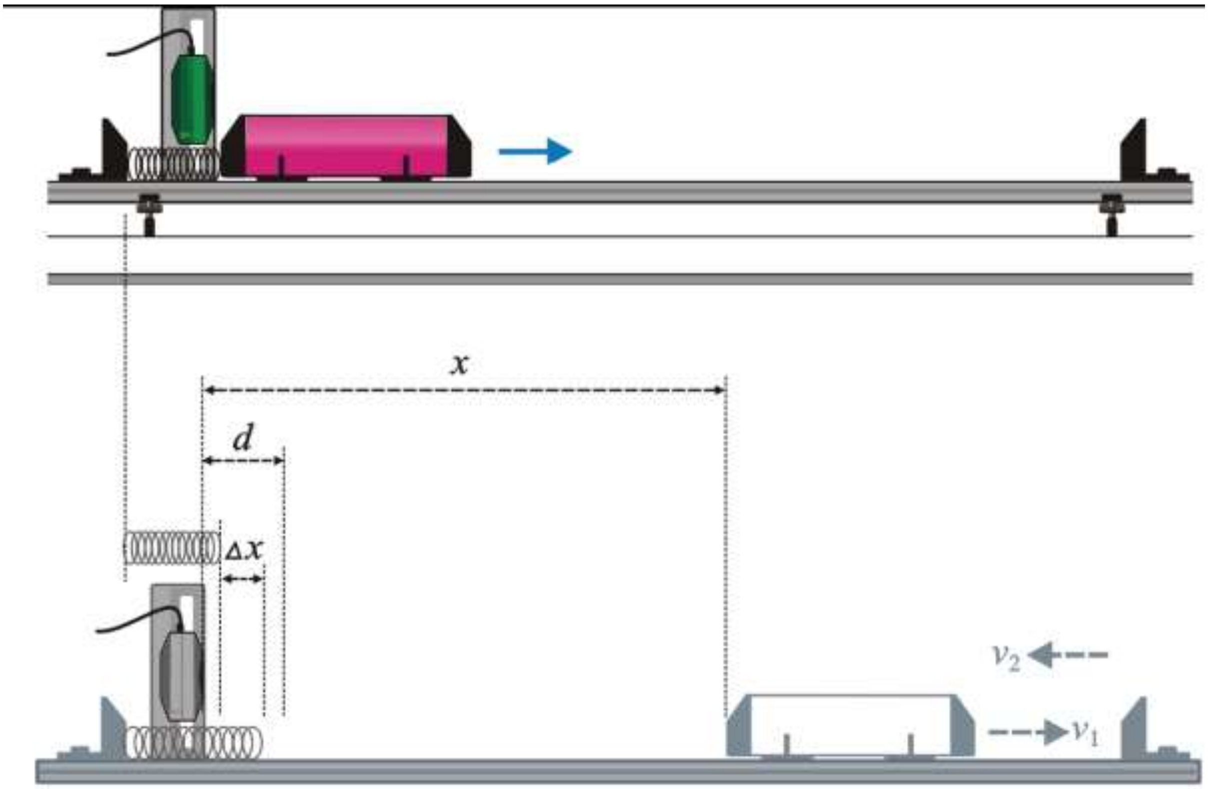
Table 3.3.1 result of analysis according to the spring's compressed length[34]




Picture 3.3.8 $x-t$ graph of the experiment according to the spring's compressed length

34 $\delta n(v)=n(v)\sqrt{\left(\frac{\delta v}{v_1}\right)^2+\left(\frac{\delta v}{v_2}\right)^2}$, $\delta n(\Delta x)=n(\Delta x)\sqrt{\left(\frac{\delta \Delta x}{\Delta x_1}\right)^2+\left(\frac{\delta \Delta x}{\Delta x_2}\right)^2}$

### 3.3.2 Motion in Which the Direction Changes: Variation of Velocity and Momentum

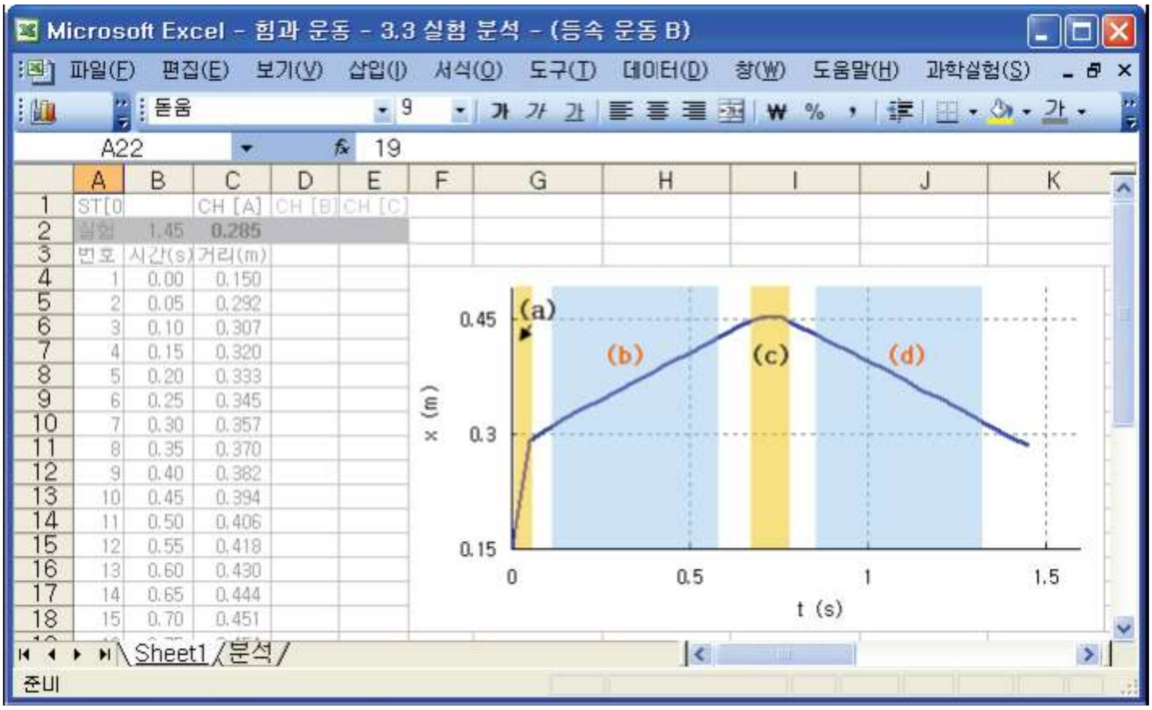



Picture 3.3.9: experiment in which the cart's direction changes: the cart springs back by the bumper and the direction is reversed.

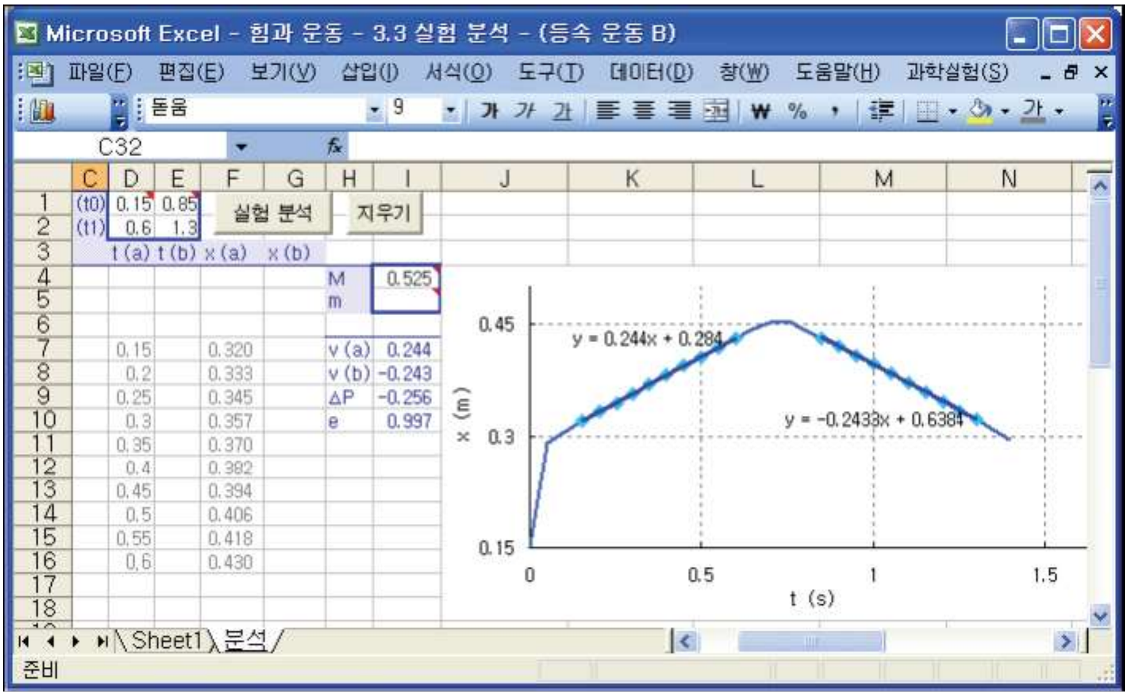
Analyze the cart's velocity with the example of picture 3.3.9. The cart is pushed by the spring at the left end of the track and it does the uniform motion, and then it springs back according to the bumper which is at the right end of the track and contains a magnet and the direction of motion is reversed. Analyze the velocity before the cart is reversed($v_1$), and the velocity after the reverse($v_2$).

Picture 3.3.10 is the result of collecting the experiment's data. Among the sections of (a), (b), (c) and (d), analyze section (b) in which the cart moves with the velocity $v_1$ and section (d) in which the cart's direction is reversed and it moves with the velocity $v_2$. Read the first time t0 and last time t1 in section (b) and the first time t0 and last time t1 in section (d) in "Sheet1". And input the time interval of section (b) in cell D1 and D2 and of section (d) in cell E1 and E2 in "Analysis" sheet. Finally, click [Experiment Analysis] button, then you can analyze the experiment[35].

[35] Use "Force and Motion(Uniform Motion B).xls" file.



Picture 3.3.10 analyzing section of experiment data collected in "Force and Motion(Uniform Motion B).xls"[36]: (b) the section that the cart moved with the velocity of $v_1$ (d) the section in which the cart's direction changes and it moves with the velocity of $v_2$



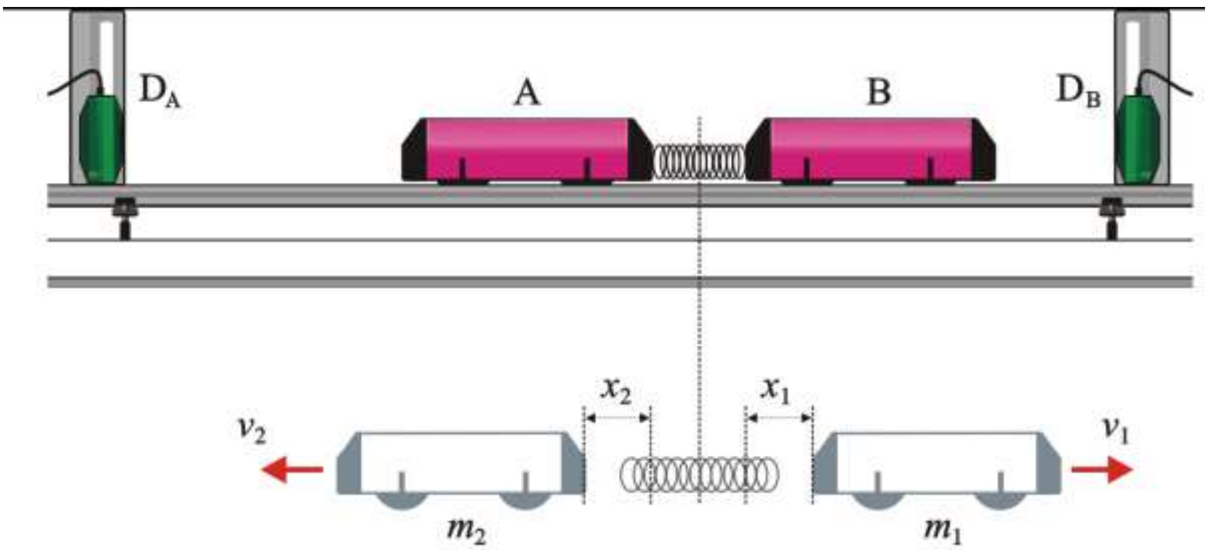
Picture 3.3.11 worksheet in which the experiment data of the uniform motion is analyzed: if you click [Experiment Analysis] button, $v_1$ and $v_2$ are analyzed and recorded in cell 17 and 18.

---

[36] Analyze section (b) and (d) except the starting process (a) and reversing process (c).

In "Analysis" sheet of picture 3.3.11, you can input the cart's mass in cell 15 and the additional mass in cell 16. When you input the mass and start the analysis, you can calculate the variation of the momentum in cell 19. In cell I10, the formula that calculates the variation of velocity $e = |v_1 / v_2|$ is applied. Section (a) is the first section that is not certain[37], and section (c) is the section in which the cart is reversed according to the magnet bumper so they need not to be analyzed.

### 3.3.3 Two Carts' Motion: Velocity and Momentum

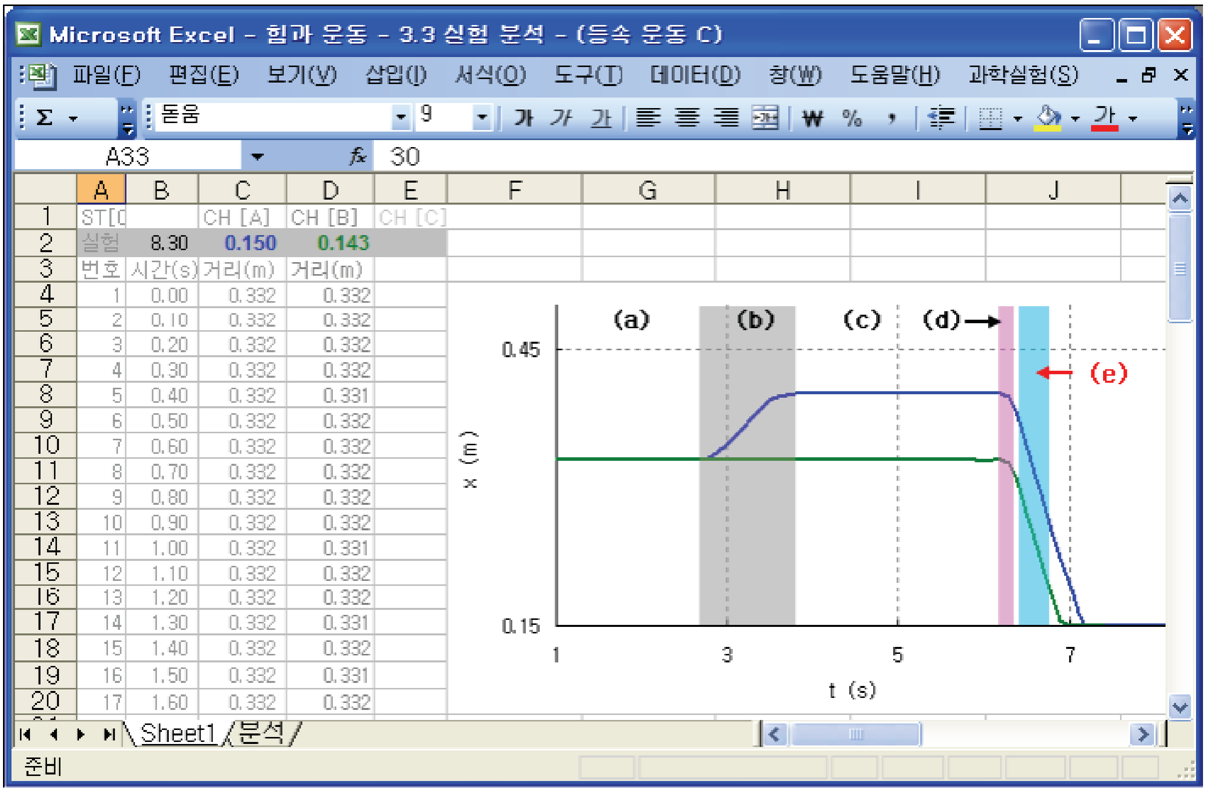



Picture 3.3.12 two carts' motion

As in picture 3.3.12, with the example of two carts' motion, calculate the carts' velocity. Set up the motion sensors at the left and right end of the track so that cart A's location is sensed by motion sensor $D_A$ and cart B's location by motion sensor $D_B$. Put the spring at the middle of the track and compress it with two carts A and B and release them, then the two

[37] As in picture 3.3.4, when the motion sensor is floating on the track and sensing the track, the data is uncertain in this section.

carts are pushed by the spring and they do the uniform motion. When the momentum is conserved completely, the ratio of two carts' mass is same as the ratio of velocity $m_1/m_2 = |-v_2/v_1|$. In actuality, the momentum can not be conserved completely because of the outer force such as the friction between the track and the carts. However, although the momentum is not conserved, two carts moves in the same physical situation, so the velocity ratio of two carts can be expressed as the ratio of mass.

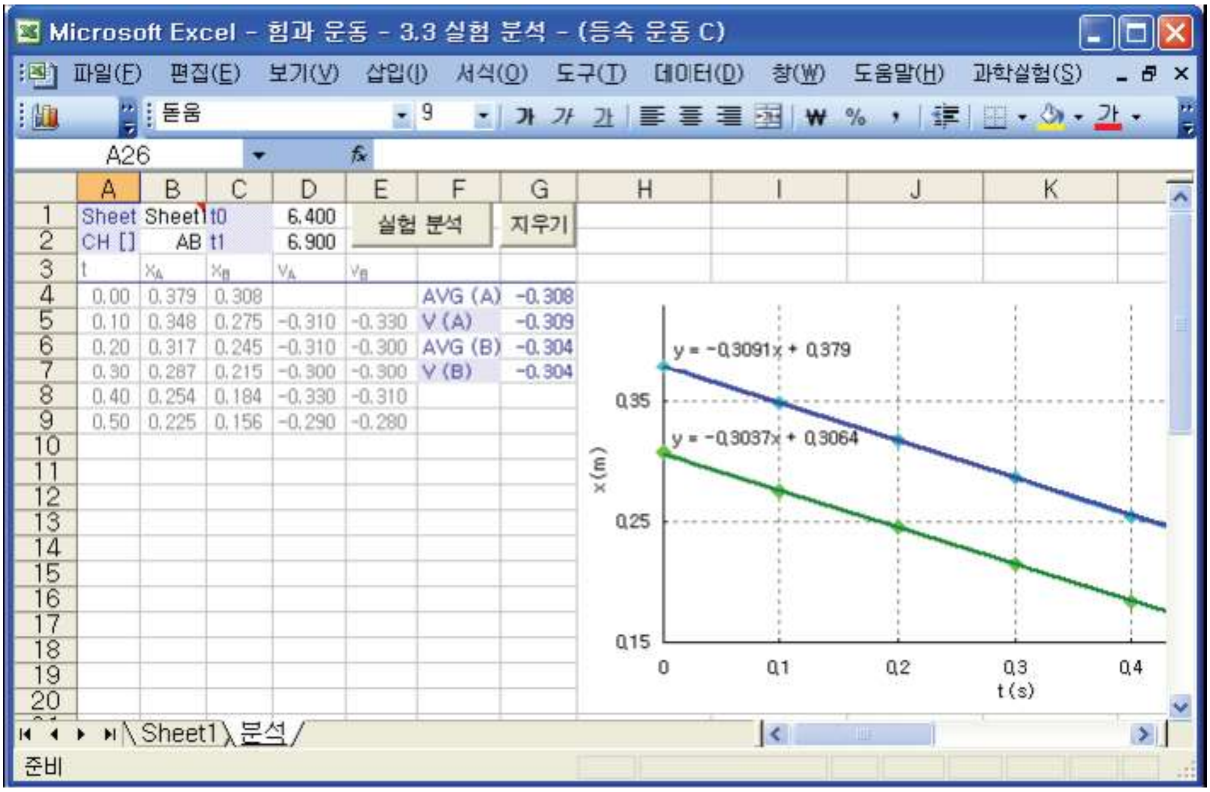



Picture 3.3.13 result data[38] and graph collected in "Force and Motion (Uniform Motion C).xls": two carts' motion

Picture 3.3.13 is the actual experiment result of the situation in picture 3.3.12. The experiment is done with connecting motion sensor $D_A$ to channel A and motion sensor $D_B$ in channel B. In section (b), if you hold fast the cart B, push the cart A toward the cart B to compress the spring and release two carts at the same time, the experiment graph as above will be acquired. (e) is the section that two carts' velocity will be analyzed. (c) is the section that the spring is compressed and two carts are not moving, and (c) is the section that two carts are released and the spring pushes two carts.

---

[38] This is the result of connecting two motion sensors to channel A and B and collecting data.




Picture 3.3.14 analysis graph of two carts' motion experiment [39] : $v_A = -0.309m/s, v_B = -0.304m/s$

In picture 3.3.13, read time t0 and t1 of section (e) in graph of Sheet1, input the first time 6.4 in cell D1 and the last time 6.9 in D2 in "Analysis" sheet, and click [Experiment Analysis] button, then you can get the analysis[40] result. In the analysis result, two carts' velocities are -0.304m/s and -0.309m/s. After calculating the error $\delta v$ and calculating the velocity, cart A's velocity is -0.304 ± 0.028m/s and cart B's velocity is -0.309 ± 0.028m/s. You can see that two carts that have same mass show same velocities within the error range[41].

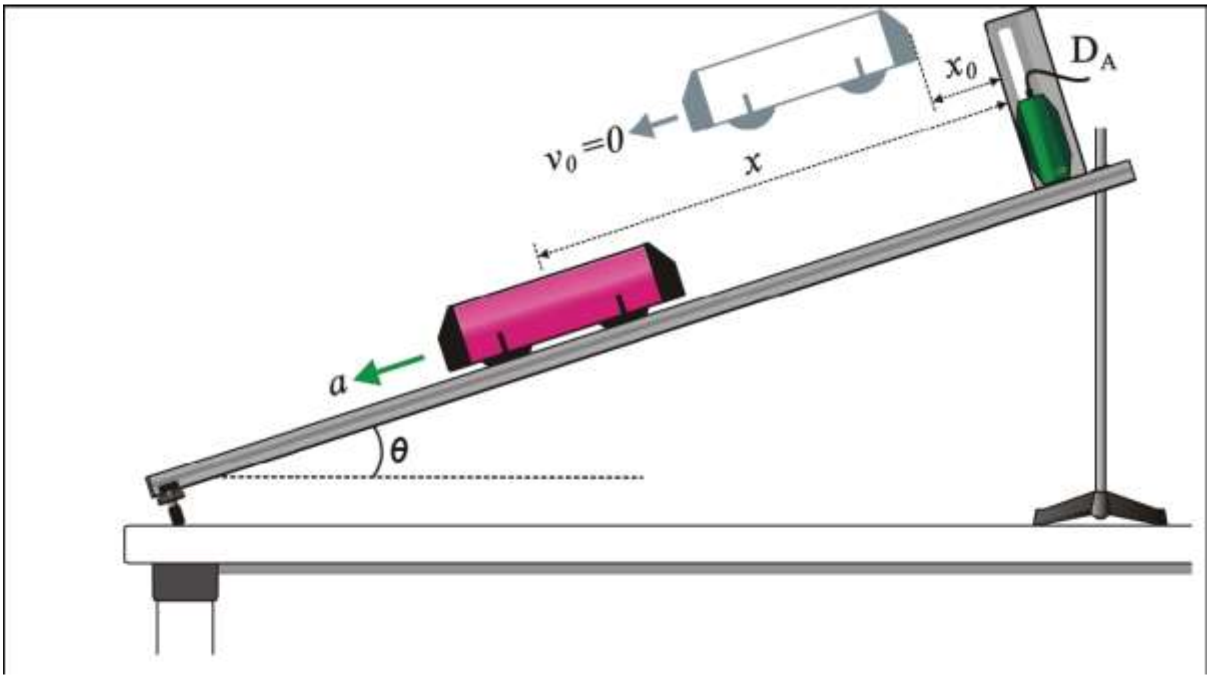
---

[39] Two $x-t$ graphs are the results of experiments done for motion sensor $D_A$ and $D_B$, and $D_A$ and $D_B$ are set up face to face, so in the result, the direction of velocity $v_A$ and $v_B$ are opposite to each other.

[40] Use "Force and Motion(Uniform Motion C).xls".

[41] $v = \frac{\Delta x}{\Delta t}$, so $\frac{\delta v}{v} \cong \sqrt{\left(\frac{\delta \Delta x}{x}\right)^2 + \left(\frac{\delta \Delta t}{t}\right)^2}$. In this case, $\delta x = \pm 0.002$ and $\delta v \approx \pm 0.028$. When the error of the distance that the motion sensor senses is $\delta x = \pm 0.002m$, the error of estimated time can be different according to the accuracy of $\delta t$'s hardware timed loop that is operated I MBL interface, but it has little effect on the error of velocity $\delta v$. The effect of $\delta t$ is very small, so when the estimating interval $\Delta t$ is the constant 0.1, if it is calculated as $\delta v \cong \sqrt{(1/0.1)^2 (\delta \Delta x)^2}$, the result is $\delta v \approx \pm 0.028$.

### 3.3.4 Motion Which Gets Constant Force: Velocity, Acceleration and Energy




Picture 3.3.15 cart's motion on a slope

When an object gets constant force during the motion, it is in the uniformly accelerated motion. As in picture 3.3.15, with the example of an experiment situation that a cart gets constant force and gets down on a slope, analyze the velocity, acceleration and energy of the cart. Make the cart stay at the location of $x_0$ [42] in front of the motion sensor $D_A$ and let it come down. When the friction is ruled out, the cart's acceleration is $a = g\sin\theta$ . The uniformly accelerated motion should be analyzed with "Force and Motion (Uniformly Accelerated Motion).xls". Refer to the download site for VBA original code of this file.

Picture 3.3.16 is the result graph of a cart's motion moving down on a slope. When you hold the cart on a slope and release it, you can get this graph so you can analyze section (a) in "Analysis" sheet. In picture 3.3.17, input the analyzing time interval t0 and t1 as the time value of $x$ axis section (a) in picture 3.3.16. AS in picture 3.3.17, if you input the time t0 in cell D1, time t1 in cell D2 and click [Experiment Analysis] button, the results will be shown in cell E5(the velocity $v_0$ at time $t_0$) and cell E4(the acceleration $a$).

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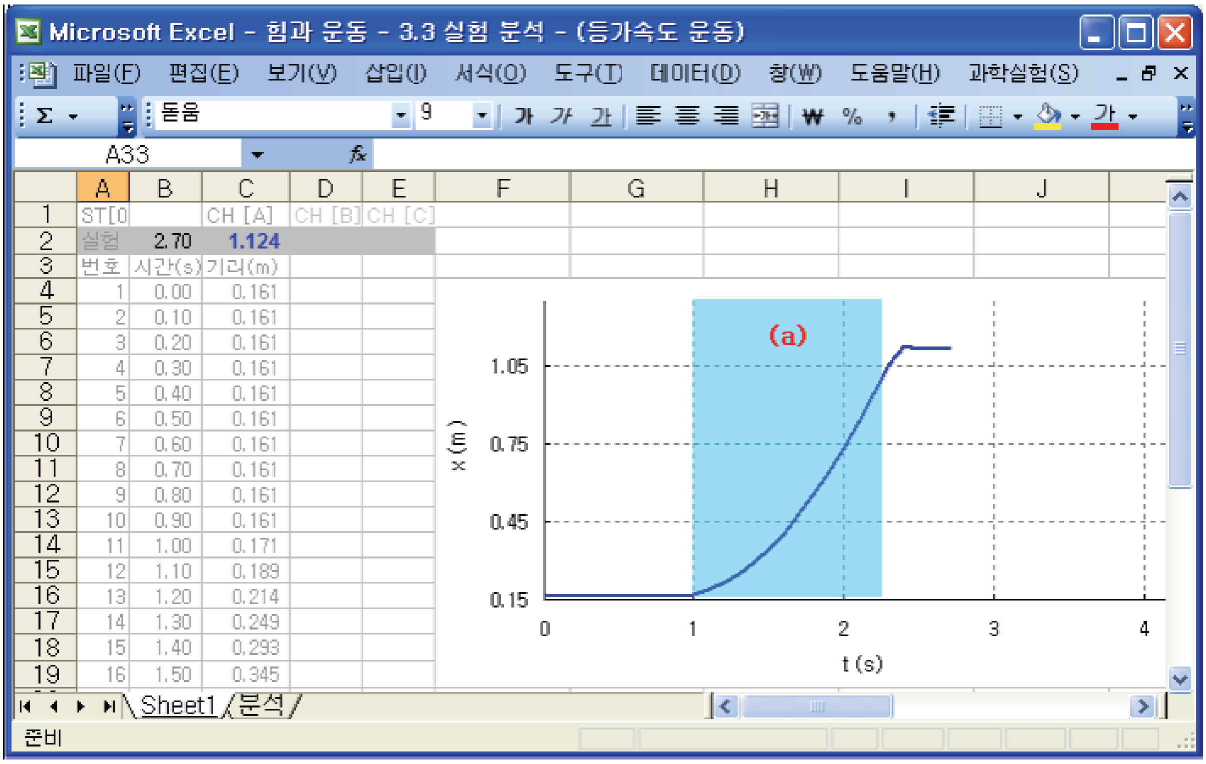
[42] When the motion sensor can sense with in the range from 0.15m to 6m, $x_0$ should be

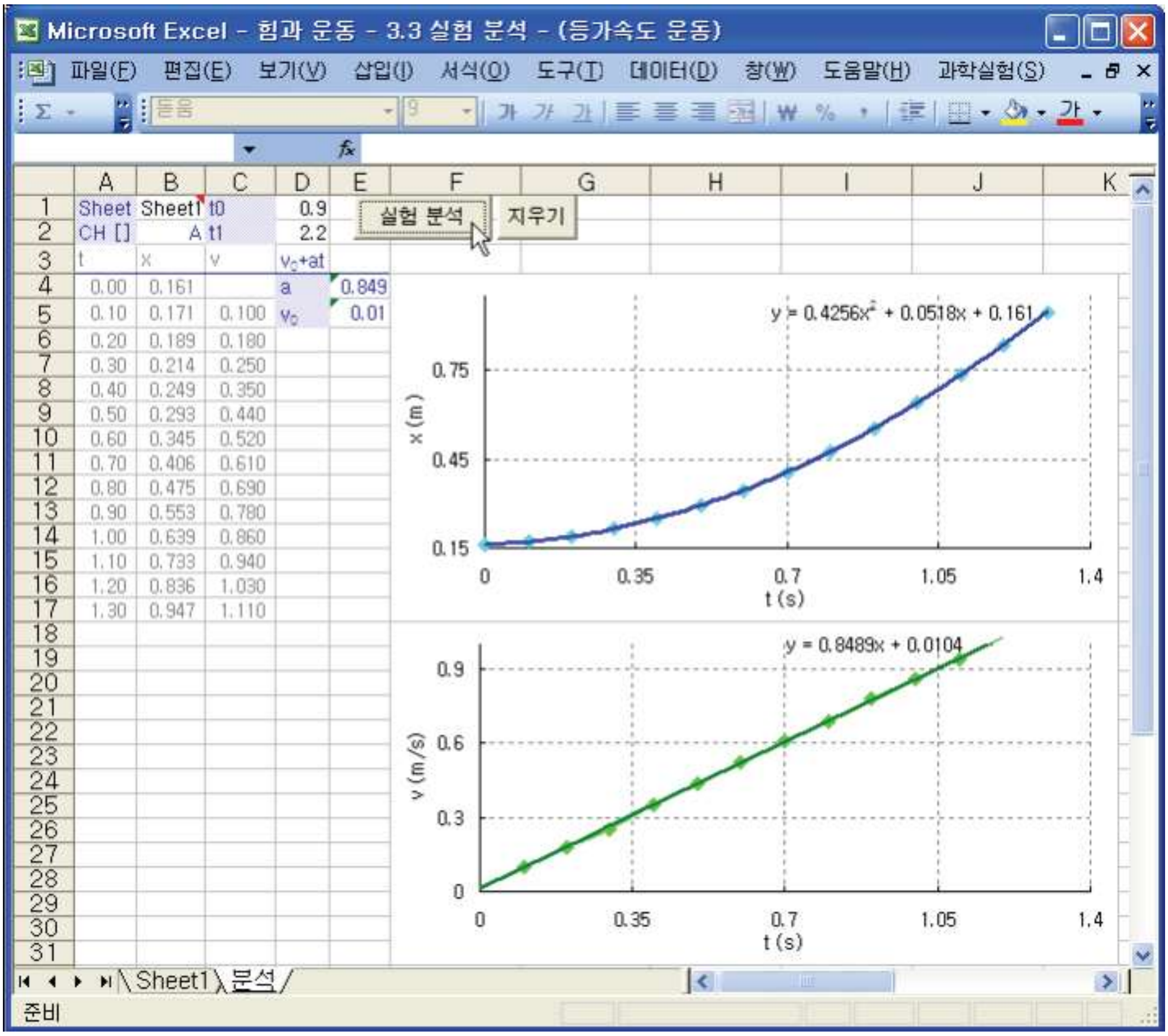



Picture 3.3.16 result data and graph of experiment in “Force and Motion(Uniformly Accelerated Motion).xls”: cart’s motion which gets constant force on a slope

In $v-t$ graph, the gradient value of the trend line formula result[43] is the acceleration $a$ and the intercept is $v_0$. In $x-t$ graph, if you differentiate the modulus of the second term of the trend line formula, you can calculate the approximate value of the acceleration $a$. This acceleration can be compared with the theoretical value. The result shown in picture 3.3.17 is acquired by calculating the acceleration 0.849m/s$^2$ when the slope’s angle is $5.0 \pm 0.2°$. In the experiment, the error of the slope’s angle is $\pm 0.2°$, so the theoretical error of the slope acceleration is $\delta g \sin\theta = \pm 0.034 m/s^2$. So the acceleration of the theoretical value is $0.854 \pm 0.034 m/s^2$. You can see that the experimental value is within the error range of theoretical value and when the estimating interval is 0.1 second, the experimental value including the error range $\delta a$[44] is $0.849 \pm 0.283 m/s^2$.

[43] To express the trend line on a chart in $x-t$ graph or $v-t$ graph, put the cursor on the graph, click the right button and choose [Add Trend Line]. For example, in $v-t$ graph, the formula of the trend line is expressed in the graph area of the chart as “y=0.8489x+ 0.0104”.

[44] $\delta a \simeq \sqrt{(1/0.1)^2 \delta v^2}$ and when $\delta v = 0.028$, $\delta a \approx 0.283$.




Picture 3.3.17 $x-t, v-t$ graphs, which are the results of analyzing the cart's motion on a slope

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### Exercise 3.3.3: Cart's Acceleration according to the Slope's Angle

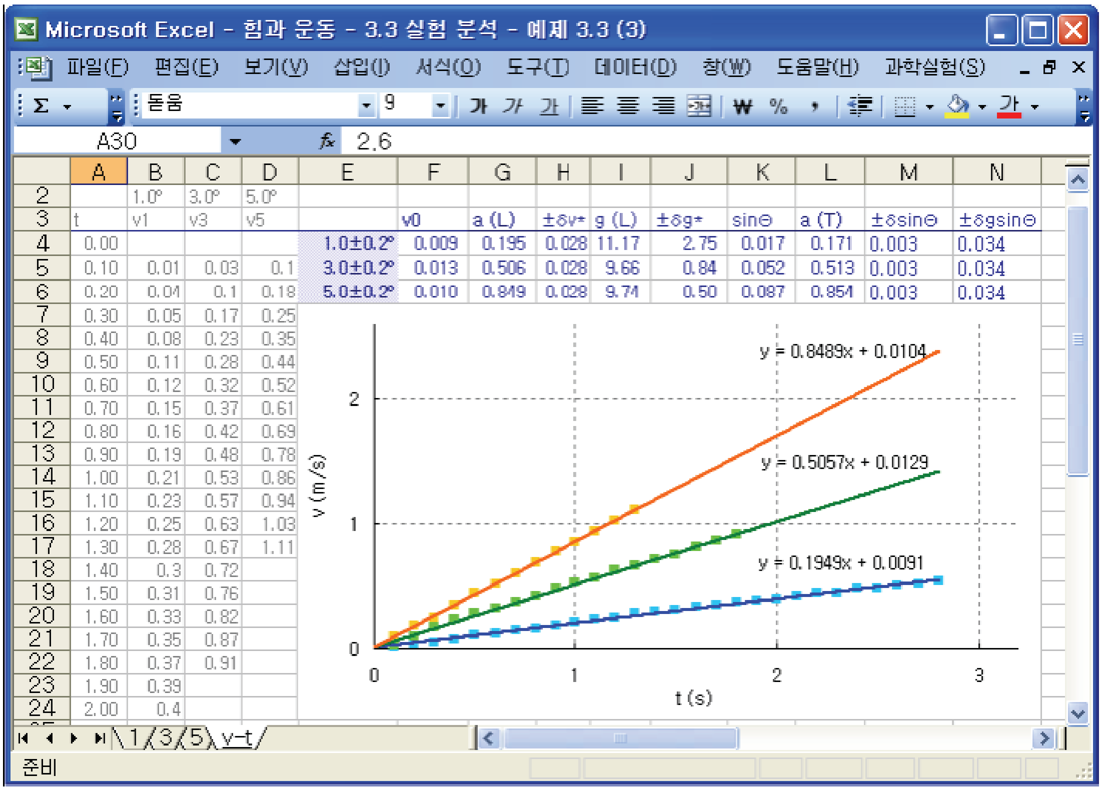
In the situation of 3.3.15, execute the experiment when the slope's angles are $1.0 \pm 0.2°$, $3.0 \pm 0.2°$ and $5.0 \pm 0.2°$.

1. Combine $v-t$ graphs according to the slope's angle in one chart[45] and express them.

---

[45] Draw charts of dispersal type using chart wizard of Excel and add to the graph the original data series of the velocity according to the slope's angle.

2. Compare and analyze the theoretical value and the result of experiment within the experimental error range and calculate the cart's acceleration and acceleration of gravity. Do not consider the frictional resistance between the track and the cart and interpret the result within the experimental error range.




Picture 3.3.18 $v-t$ graph, which is the result of analyzing the cart's motion according to the slope's angle: the slope's angles are $1.0 \pm 0.2°$, $3.0 \pm 0.2°$ and $5.0 \pm 0.2°$

Picture 3.3.18 is the $v-t$ graph and the result of analyzing the experiment data of the slopes that have angles of $1.0 \pm 0.2°$, $3.0 \pm 0.2°$ and $5.0 \pm 0.2°$. The experiment result that is compared with the theoretical value and analyzed within the experimental error range is as table 3.3.1. The acceleration a(L) is the experimental value, and a(T), $\delta g \sin g\theta$ are the theoretical values.

| | a(L) | g(L)±$\delta g$* | ±$\delta$sin$\theta$ | a(T) | ±$\delta g$sin$\theta$ |
|---|---|---|---|---|---|
| 1.0±0.2° | 0.195 | 11.17±2.75 | 0.003 | 0.171 | ±0.034 |
| 3.0±0.2° | 0.506 | 9.66±0.84 | 0.003 | 0.513 | ±0.034 |
| 5.0±0.2° | 0.849 | 9.74±0.50 | 0.003 | 0.854 | ±0.034 |

Table 3.3.1 result of the cart's motion according to the slope's angle

In table 3.3.1, $\delta g*$ is the error caused by the acceleration of gravity and it can be calculated as follows.

$$\delta \sin\theta = \cos\theta \cdot d\theta$$

$$\delta g = g\sqrt{\left(\frac{\delta a}{a}\right)^2 + \left(\frac{\delta \sin\theta}{\sin\theta}\right)^2}$$

In this case, when the estimating interval is 0.1 second, $\delta a = 0.28$, so when calculating the velocity with the gradient of the trend line in $x-t$ graph by expanding the estimating range of time as 1.0 second[46] and heightening the resolution of the analysis , it can be calculated as $\delta a \approx 0.028$ and the result is $\delta g*$ of table 3.3.1. The analysis result of table 3.3.1 can be acquired by inputting the calculating formula in worksheet of Excel.

IN the result of table 3.3.1, When the estimating interval is 0.1 second, $\delta v = 0.028$, so the experimental error of the acceleration is $\delta a \approx 0.28$[47]. This value is acquired from the distance estimating error of $\pm 0.002m$. You can see that the experimental value of the acceleration a(L) is within the range of the theoretical value a(T)$\pm \delta g \sin\theta$. For example, when the slope's angle is $5°$, the experimental value of the acceleration is 0.506 and this it within the theoretical value of the error range $0.513 \pm 0.034$.

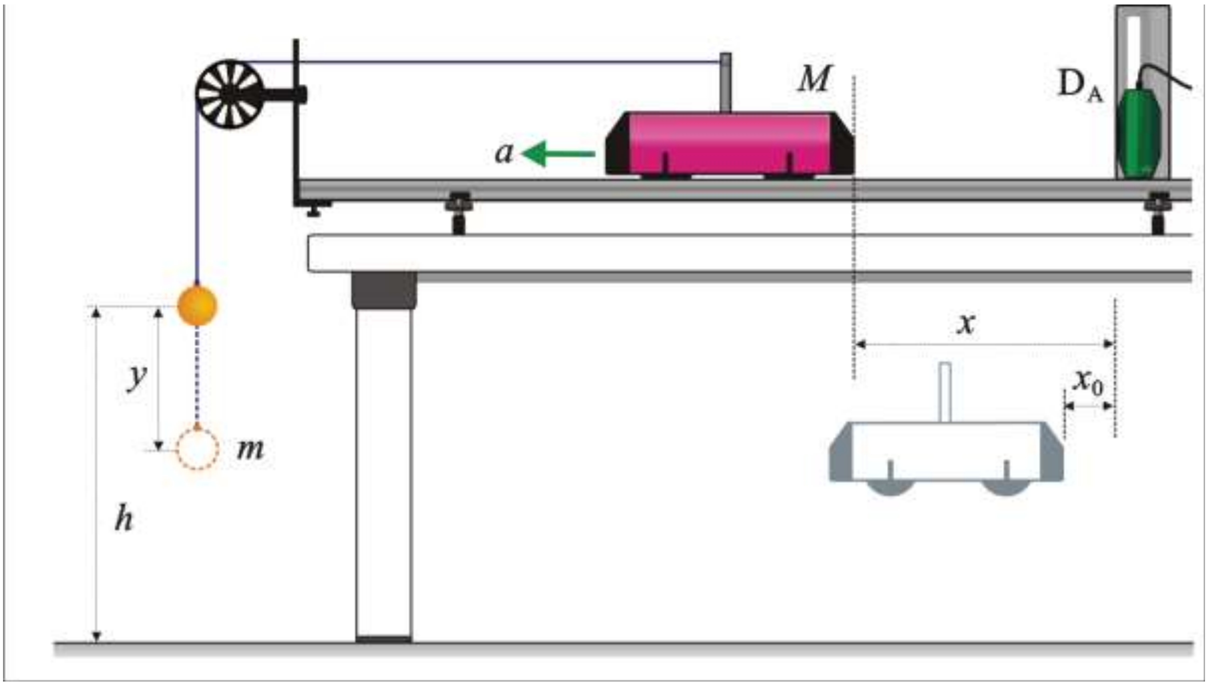
### Exercise 3.3.4: Cart's Acceleration according to the Pendulum's Falling

1. Picture 3.3.19 is the situation that a cart of mass M is moving by a pendulum of mass m. The cart's mass M is 0.525kg, the pulley's mass $M_r$ is 0.0061kg and the radius r is 0.032m. When the pendulum's masses are 0.05kg and 0.1kg and the cart's mass is 1.026kg, calculate the cart's acceleration for each situation by experiment and calculate the modulus of friction between the cart's wheels and the track.

2. Draw the relationship graph between the cart's potential energy V, kinetic energy T and mechanical energy E and see if the cart's total energy is conserved.

---

[46] To heighten the resolution of the analysis by narrowing the error range, make the estimating range bigger so that the values to speculate and analyze will be increased.
[47] To make the experimental error of the acceleration, choose other sensors that have smaller value of distance estimating error or other experiment methods.




Picture 3.3.19 cart's motion according to the pendulum's falling

<u>Explanation 1:</u>

The pulley's moment of inertia[48] $I = 1/2 M_r r^2$ is as follows.

$$I = \frac{1}{2} \times 0.0061 \times 0.032^2 = 3.09 \times 10^{-6} \mathrm{kg \cdot m^2}$$

Calculate the modulus of friction $\mu$ with formula (3.1.12). The acceleration $\ddot{x}$ should be calculated with the experiment. The modulus of friction $\mu$ is like below.

$$\mu = \frac{mg - \left(M + m + \frac{I}{r^2}\right)\ddot{x}}{Mg}$$

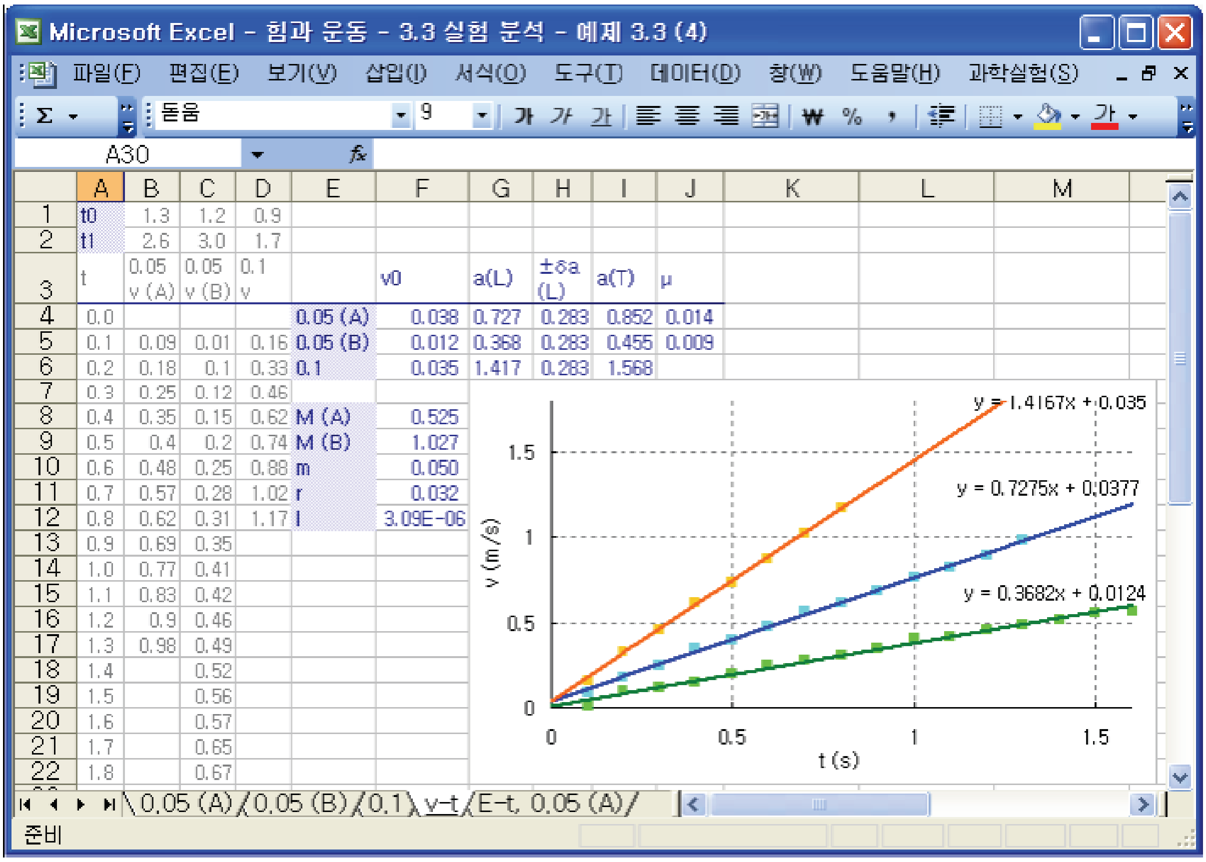
Table 3.3.2 is the real experiment result of the cart's motion by a pendulum. a(T) * is the theoretical value calculated with $\mu = 0, I = 0$. When the pendulum's mass gets bigger, the acceleration $\ddot{x}$ increases, and when the cart's mass gets bigger, the modulus of friction also gets bigger. Also, you can see that the experimental value shows meaningful values within the experimental error range, compared with the theoretical value.

---

48 It can be calculated differently according to the pulley's shape.

| m | M | v0 | a(L) | ±δa | a(T)* | μ |
|---|---|---|---|---|---|---|
| 0.05 | 0.525 | 0.038 | 0.727 | ±0.283 | 0.852 | 0.014 |
| 0.05 | 1.026 | 0.012 | 0.368 | ±0.283 | 0.455 | 0.054 |
| 0.1 | 0.525 | 0.035 | 1.417 | ±0.283 | 1.568 | |

Table 3.3.2 result of the cart's motion experiment according to the pendulum's falling




Picture 3.3.20 $v-t$ graph of the cart's motion experiment according to the pendulum's falling

<u>Explanation 2:</u>

First, execute the experiment that estimates a cart's location with a motion sensor in "Sheet1" of "Force and Motion(Uniformly Accelerated Motion).xls". For each experiment, analyze the result in "Analysis" sheet as in picture 3.3.16 and 3.3.17 and calculate $x(t), v(t)$ and $v_0$ between time $t_0$ and $t_1$. Second, as in table 3.3.2 and picture 3.3.20, calculate the modulus of friction $\mu$ . Lastly, with formula (3.1.14), from the experimental value of $x(t), v(t)$ , calculate the kinetic energy T(t), potential energy V(t), loss energy due to the frictional resistance R(t) and total energy E(t) as in table 3.3.3.

| t | x | v | T(t) | V(t) | R(t) | E(t) |
|---|---|---|---|---|---|---|
| 0 | 0.205 | - | - | - | - | - |
| 0.1 | 0.214 | 0.090 | 0.005 | 0.344 | 0.015 | 0.364 |
| 0.2 | 0.232 | 0.180 | 0.014 | 0.335 | 0.017 | 0.365 |
| 0.3 | 0.257 | 0.250 | 0.024 | 0.323 | 0.019 | 0.365 |
| 0.4 | 0.292 | 0.350 | 0.043 | 0.306 | 0.021 | 0.370 |
| 0.5 | 0.332 | 0.400 | 0.055 | 0.286 | 0.024 | 0.365 |
| 0.6 | 0.380 | 0.480 | 0.077 | 0.263 | 0.027 | 0.367 |
| 0.7 | 0.437 | 0.570 | 0.106 | 0.235 | 0.031 | 0.372 |
| 0.8 | 0.499 | 0.620 | 0.124 | 0.204 | 0.036 | 0.364 |
| 0.9 | 0.568 | 0.690 | 0.152 | 0.171 | 0.041 | 0.363 |
| 1.0 | 0.645 | 0.770 | 0.187 | 0.133 | 0.046 | 0.366 |
| 1.1 | 0.728 | 0.830 | 0.216 | 0.092 | 0.052 | 0.361 |
| 1.2 | 0.818 | 0.900 | 0.252 | 0.048 | 0.059 | 0.359 |
| 1.3 | 0.916 | 0.980 | 0.297 | 0 | 0.066 | 0.363 |

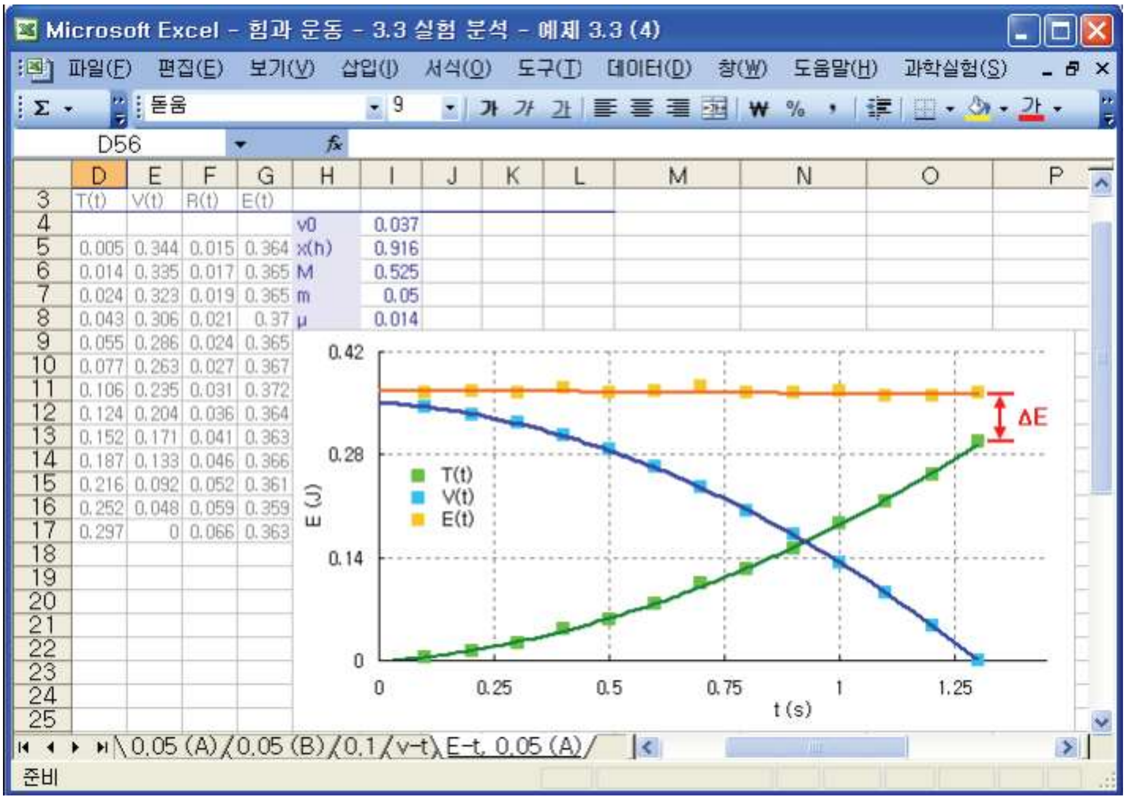
Table 3.3.3 analysis result of the cart's motion experiment according to the pendulum's falling (1)

Formula that calculates the results of table 3.3.3 in Excel worksheet is as follows. For example, in order to make T(t) be calculated in column D, input formula as in table 3.3.4. In table 3.3.4, "(Experiment Analysis Value)" is the result of analysis done in "Force and Motion(Uniformly Accelerated Motion).xls". The masses of the cart and the pendulum M and M are estimated with the electric scale. As in the way of table 3.3.4, write the calculating formula in a new worksheet and analyze the experiment result. Picture 3.3.21 is the result of analyzing the relationship graph of total energy[49] totally in this way.

[49] Draw charts of dispersal type using chart wizard of Excel and add the original data series to the graph.

| Cell, Column | | Formula, Value |
|---|---|---|
| v0 | I4 | (Analyzed value) |
| x(h) | I6 | =+MAX(B4:B200) |
| M | I7 | (Estimated value) |
| m | I8 | (Estimated value) |
| μ | I9 | (Analyzed value) |
| x | B | (Analyzed value) |
| v | C | (Analyzed value) |
| T(t) | D | =1/2*($I$6+$I$7)*(C5+$I$4)^2 |

Table 3.3.4 ways of inputting formula and value in cell of Excel worksheet[50]



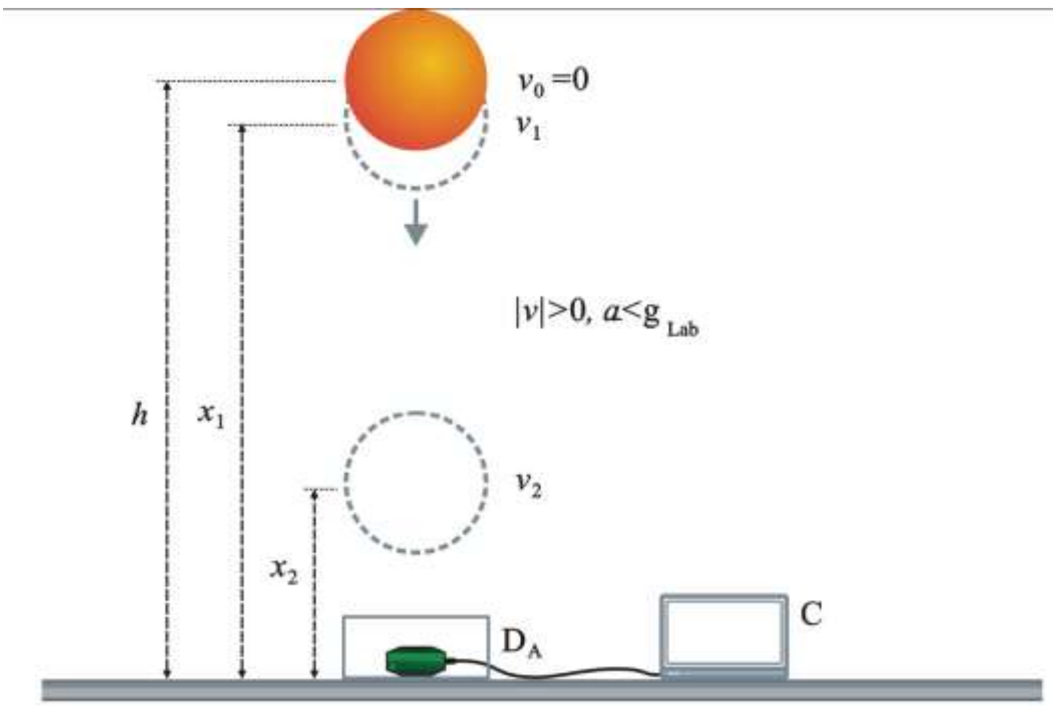

Picture 3.3.21 E-t graph, which is the result of the cart's motion experiment according to the pendulum[51]

[50] Make a new worksheet and write them down. For example, the result of writing in the sheet named "E-t, 0.05 (A)" is picture 3.3.20.

[51] If the loss energy according to the friction( $R(t) = \Delta E$ ) is not calculated, the graph of total energy E(t) will be tilted lower than the horizontal line and it will be drawn in the shape of meeting at the end of the kinetic energy T(t).

### Exercise 3.3.5: Ball's Falling Motion

1. As in picture 3.3.22, when you make a basketball fall with the initial velocity $v_0 = 0$, calculate the energy loss caused by the air resistance and the drag constant.



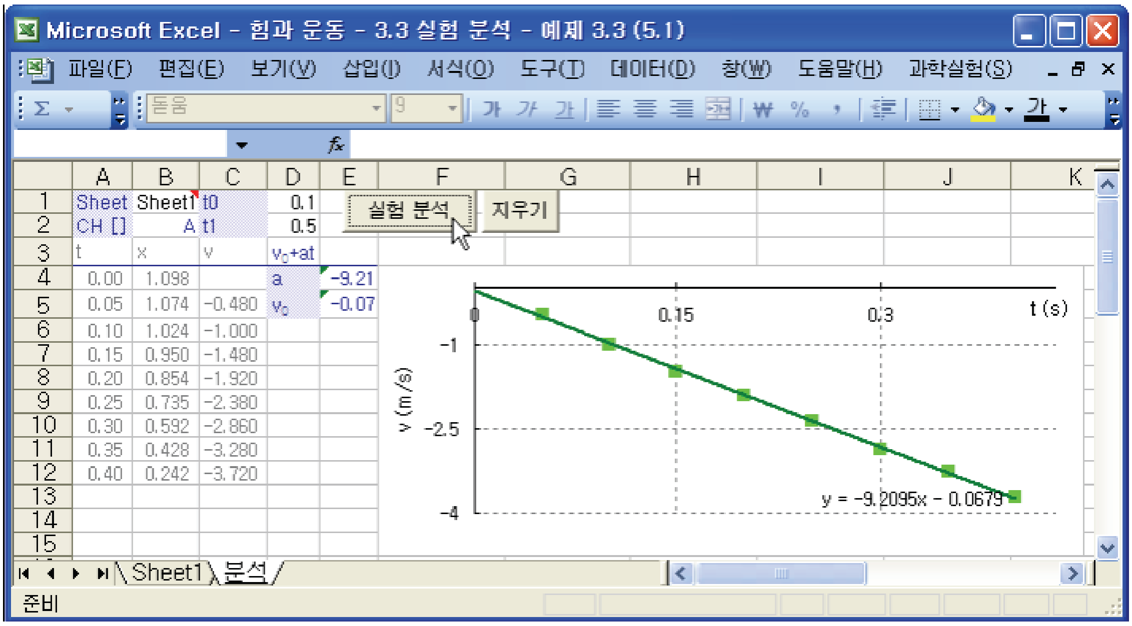

Picture 3.3.22 ball's falling motion experiment

Explanation:

When the basketball is falling, the force it gets by the air resistance is in proportion to the square of velocity[52]. The loss energy R according to the air resistance is same as the difference between the basketball's potential energy V and kinetic energy T. Picture 3.3.23 is the result of the experiment that make a basket ball of 0.5kg mass fall. In the result, when the basketball falls from $x_1$ to $x_2$, the acceleration is $-9.12\, m/s^2$ and the velocity is $v_2 = -3.72\, m/s^2$ .

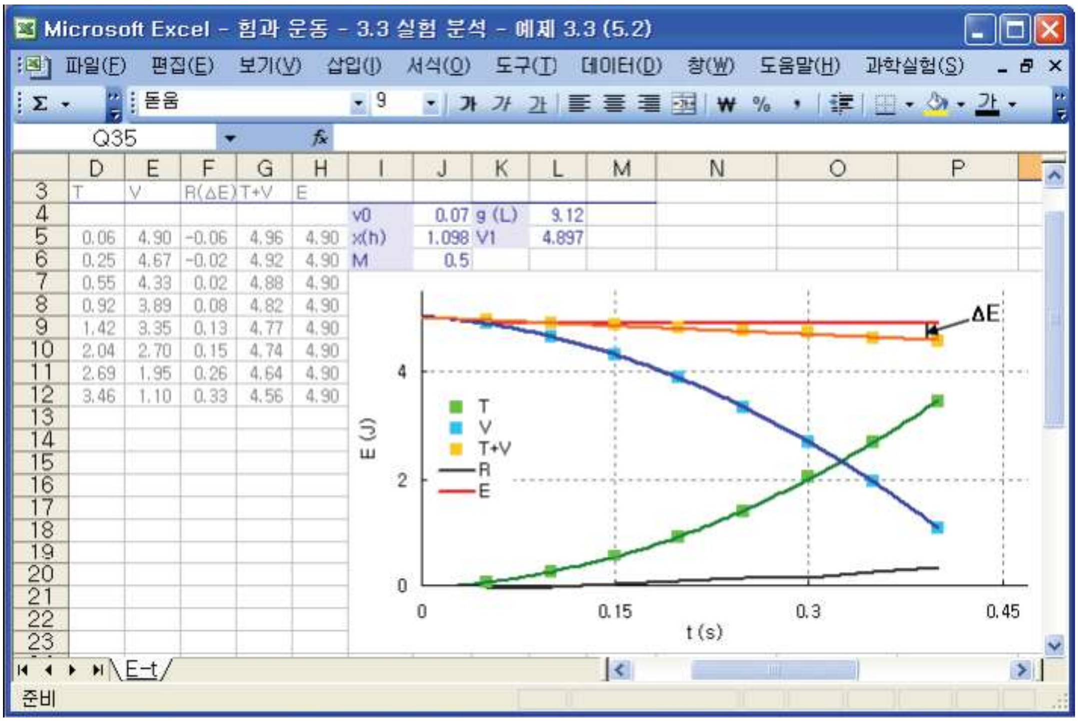
[52] When the force caused by the air resistance operates as $-cv^n$, if the falling velocity is small experimentally, $n \cong 1$, and if it is big, $n \cong 2$. In case of a basketball, when the terminal velocity is calculated the force that corresponds to n=2(quadratic force) works. When n=2, the force and the terminal velocity can be expressed as follows.

$$F = mg - cv^2 = mg\left[1 - \frac{c}{mg}v^2\right],\ v_T = \left(\frac{mg}{c}\right)^{1/2}$$




Picture 3.3.23 v-t graph of the basketball's falling motion experiment

This acceleration can be different according to the basketball's velocity. In this case, if the drag constant c[53] is calculated, $c = 0.0246Ns^2/m^3$. Also, the loss energy according to the air resistance from $x_1$ to $x_2$ is $R(\Delta E) = 0.33J$.

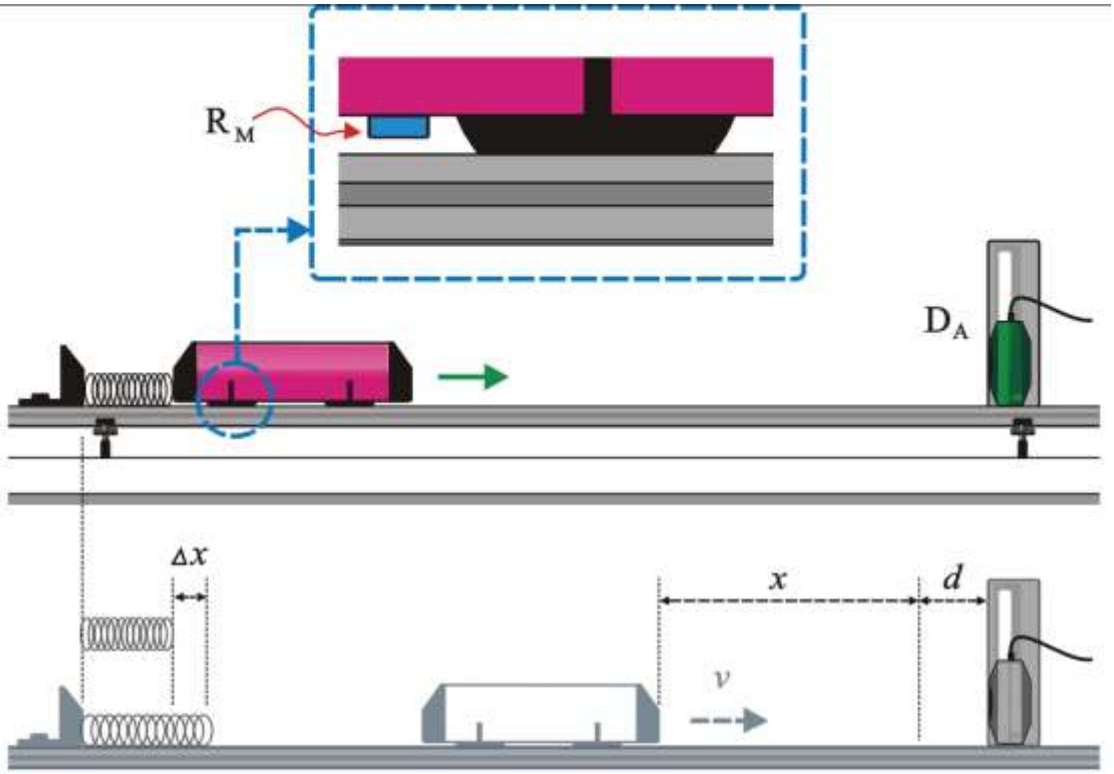



Picture 3.3.24 E-t graph of the basketball's falling motion experiment

53 $c = m(g_{Lab} - a_t)/v_t^2$

### 3.3.5 Motion in Which the Force Changes: Location, Velocity and Energy

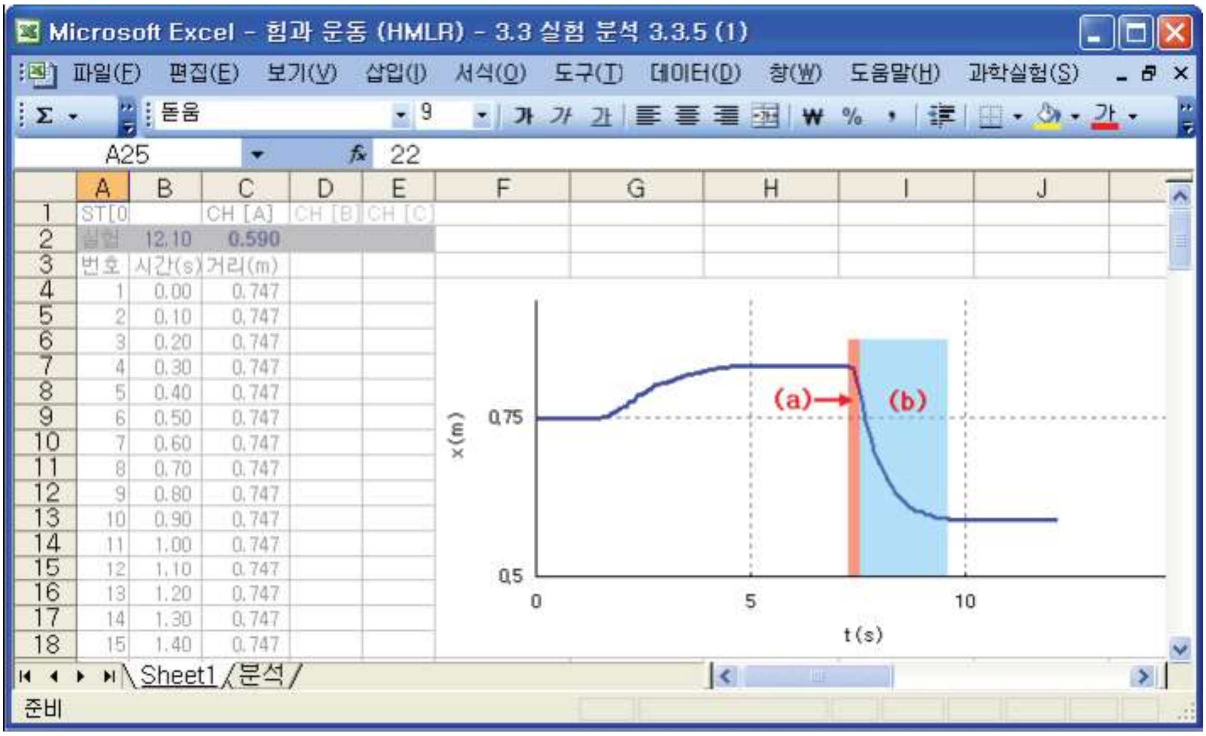
As in picture 3.3.25, move a cart that a neodymium magnet $R_M$ is attached on a track. As the cart moves, a force is generated according to the magnet in the direction of disturbing the cart's motion[54]. This force is in proportion to the velocity and changes regularly. Analyze the cart's velocity and calculate the energy loss according to the resistance $dE/dt$.




Picture 3.3.25 experiment of a cart's motion that gets damping resistance according to the magnet

Picture 3.3.26 is the $x-t$ graph of the experiment done by attaching two magnets at the bottom of the cart. As in picture 3.3.25, when the spring is compressed as much as $\Delta x$ and released, the cart moves on the track. At this moment, as the cart moves and it gets the magnet's resistance, so it shows the damping motion that the cart's velocity decreases exponentially. Picture 3.3.27 is the result of analyzing the result of picture 3.3.26 in "Analysis" sheet of "Force and Motion(HMLR).xls" file.

[54] When the cart moves, the changes caused by the magnet in the magnetic field make the eddy current on the surface of the track, and this current makes the magnet field in the opposite direction of the magnet's motion so it disturbs the cart's motion. In chapter 5, a cart's oscillation that gets damping resistance is introduced.




Picture 3.3.26 result of a cart's motion that gets force which changes regularly

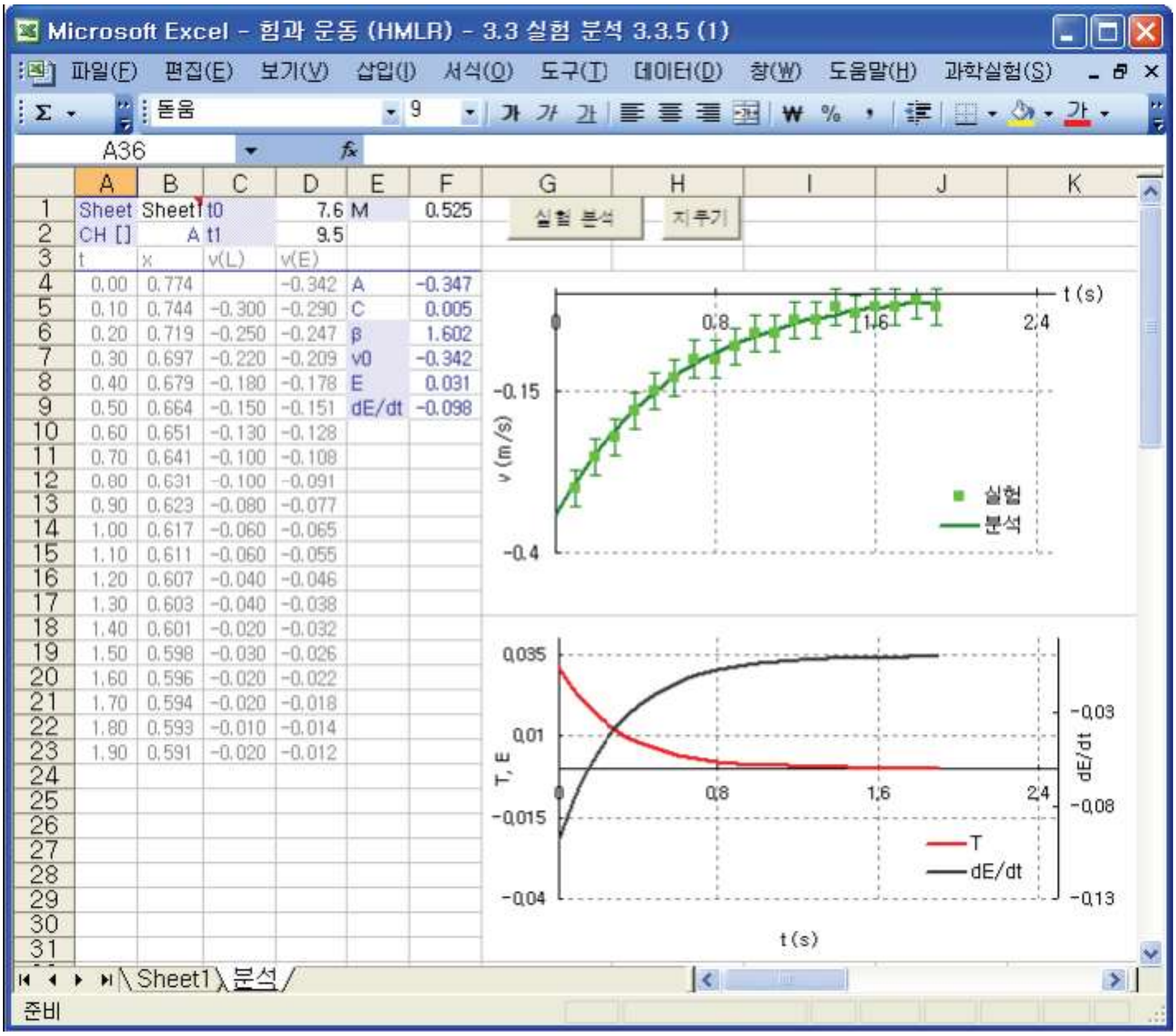
In section (b) of picture 3.3.26, from the time t0=5.9 to t1=8.9[55] in which the cart moves with the resistance, when the cart's mass is 0.525kg, the initial velocity $v_0 = -0.383 m/s$ and the kinetic energy is 0.0385J. The cart's velocity $v(t)$ can be expressed as below.

$$v(t) = -0.383e^{-0.490t}$$

The cart's velocity $v_0$ is near to the velocity of the cart when the spring is compressed and released to push the cart[56]. At this moment, the cart's total energy E is same as the kinetic energy T and you can read the value in cell F8 in picture 3.3.25. The value of cell F9 is the ratio of the energy loss during the cart's motion dE/dt and this value is different according to the damping resistance of the velocity.

[55] Choose the section between t0 and t1 properly, in which the exponential fitting graph fits best within the error stick of the experiment data, and read it.

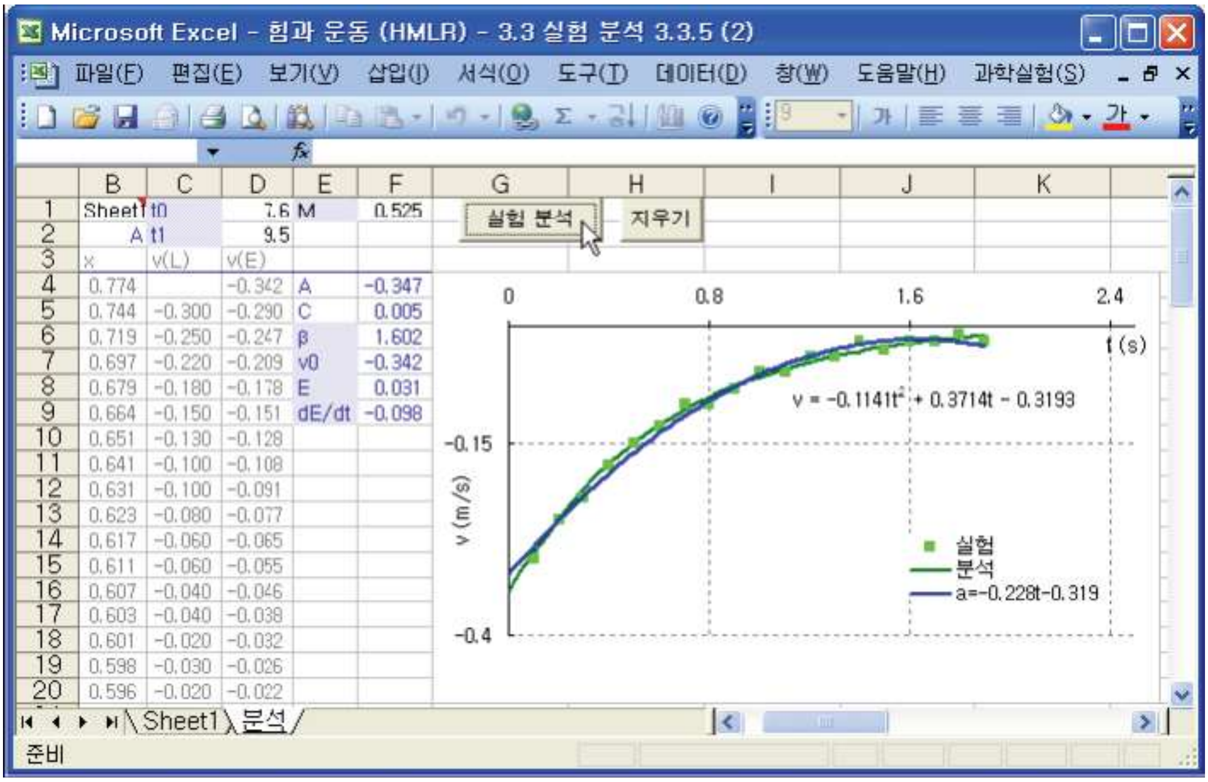
[56] This can be different according to the moment that the analysis begins, that is, t0. When reading t0 and t1, in picture 3.3.24, you should rule out the short section (a) which is the beginning part of the motion and in which the cart gets force according to the spring and the last section that the cart finishes moving and stops.




Picture 3.3.27 graph of analyzing the cart's motion according to the damping resistance

Picture 3.3.27 is the result of analyzing the cart's location and velocity with the exponential curve fitting[57]. When you click [Experiment Analysis] button in "Analysis" sheet, you can analyze the damping constant $\beta$, the initial velocity $v_0$, energy E and dE/dt and you can see the result of the graph. Theoretically, you can see that when the cart gets the force $-bv$ that is in proportion to the velocity and when the cart's velocity is expressed as $v_0 e^{-\frac{b}{m}t}$ just as formula (3.1.16), the real experiment result is the same. At this moment, if the time $t_0 = 0$, the acceleration $a_0 = -\beta v_0$. The graph's result is analyzed within the velocity's error range $\delta v = \pm 0.028$.

[57] When the $v-t$ graph is in the shape of the quadratic curve and gets the force in proportion to the velocity, you should calculate the formula of curve fitting to which the exponential function about $v = v_0 e^{-\frac{b}{m}t}$ is applied.




Picture 3.3.28 $v-t$ graph to which the quadratic polynomial trend analysis is added

Picture 3.3.28 is expressed by adding the quadratic polynomial trend analysis about the experiment data of "Analysis" sheet. The formula result of the trend analysis is $v = -0.114t^2 + 0.371t - 0.319$ and $a = \dfrac{dv}{dt} = -0.228t - 0.319$. Experimentally, you can see that the result of the quadratic polynomial trend analysis is same as the result of the exponential analysis within the experiment error range. When you see the result of the quadratic polynomial analysis, the acceleration $a$ is in the form of the time's linear function and you can see that it is the motion that the acceleration's rate of change is constant according to the time. You should research the difference between the theory and the experiment of the exponential analysis and the quadratic polynomial analysis. Also, as explained in chapter 2, you can use the modeling analysis. Like this, according to the experimental situations, you can analyze the motions differently[58]. Therefore, I hope that these experiments and analysis process suggested in book are helpful for you to learn high level physics experiments.

[58] There can not be only one way to explain physical phenomenon, and there are various creative ways of solving problems in physics education.

# 3.4

# Experiment: Velocity and Momentum

## 3.4.1 Experiment Outline

Using the first dimensional track and a cart, let's estimate the cart's time and location in a motion with constant direction, a motion that the direction changes oppositely and a motion that two carts do the uniform motion. Using these experiment data, let's analyze the cart's velocity and study the momentum and the change of momentum

### Goal

Learning the concept of velocity in uniform motion and understanding the momentum and the change of momentum

### Required Equipments

| | | | |
|---|---|---|---|
| Electronic scale | 1 | Pendulum(0.5kg) | 1 |
| Motion sensor | 2 | Pendulum(50g) | 2 |
| Springs(0.142m, 22.895N/m) | 1 | Double stick tape | 1 |
| Neodymium magnets | 4 | Holding tape | 1 |

### 3.4.2 Experiment A: Motion with Constant Direction

---

**Experiment Prediction:**

1. Make the cart do the uniform motion by pulling and releasing the elastic cord. How will the cart's velocity change according to the elastic cord's pulled length?

2. When the spring is compressed and released, how will the pushed cart's velocity change according to the force gotten from the spring? How will it change according to the cart's mass?

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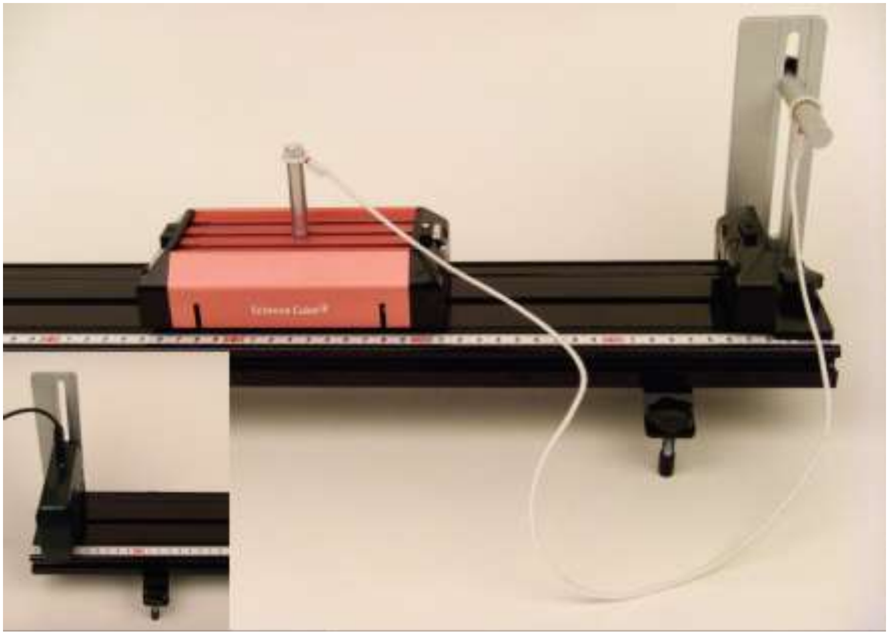
**Experiment Process A: Cart's Motion Pulled by Elastic Cord**



Picture 3.4.1 cart's motion that is pulled by the elastic cord[59]

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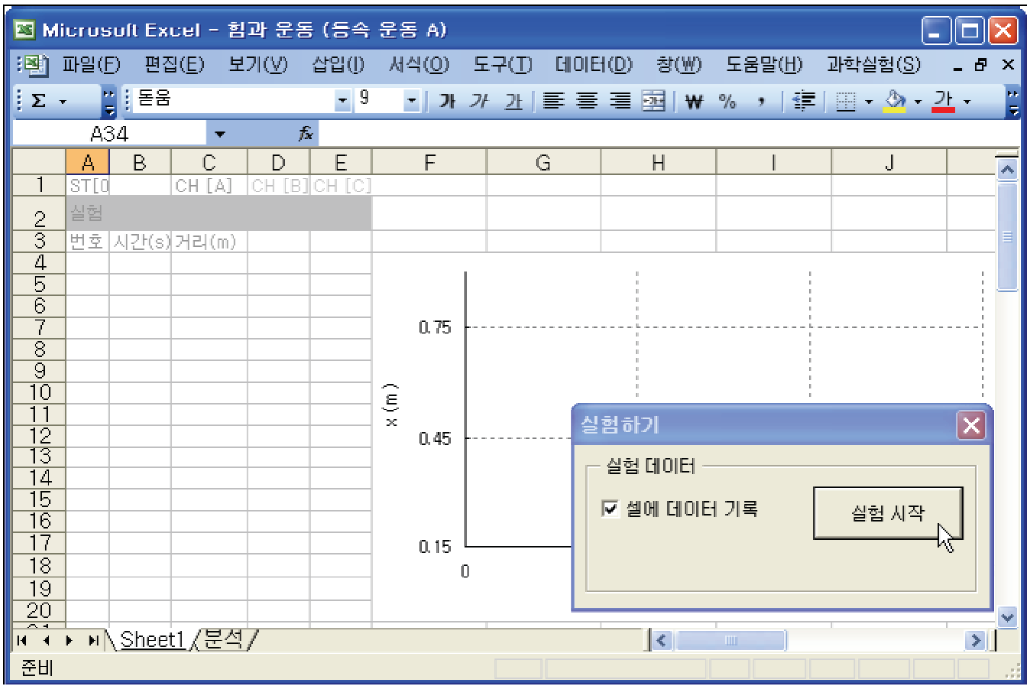
[59] Set up the equipments as in picture 3.4.1 so that the elastic cord does not disturb the cart's motion.

1. Set up the motion sensor at the left end of the track as in picture 3.4.1 and prepare so that the elastic cord can make the cart move.

2. Connect the motion sensor and the computer, open “Force and Motion(Uniform Motion A).xls” file and execute the experiment.

a. Open “Sheet1” in Excel and open [Science Cube]-[Experiment Setting] window in worksheet menu to set up the estimating interval as 0.1 second, and the experiment time as 5 seconds.

b. As in picture 3.4.2, open [Science Cube]-[Experiment] window in worksheet menu of “Sheet1” and click [Start Experiment] button.




Picture 3.4.2 experiment with opening “Sheet1” of Excel workbook[60]: If you click [Start Experiment] button, data will be collected in the sheet.

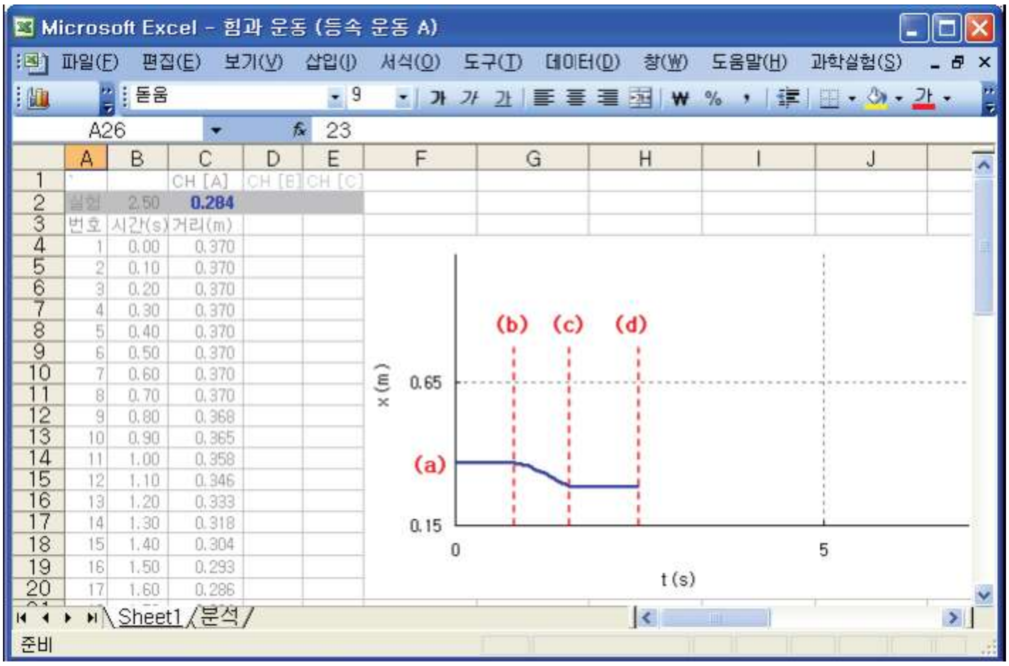
c. If you click [Start Experiment] button, as the graph section (a)-(b) in picture 3.4.3, the experiment data will be collected within the sheet of workbook.

---

[60] Use “Force and Motion(Uniform Motion A).xls” file.

d. If you move the cart slowly toward the motion sensor, the data will be collected as the graph section (b)-(c) in picture 3.4.3. Section (c)-(d)[61] is the state of staying after moving the cart.

e. Release the cart slightly at the point of (d) in picture 3.4.3 and make the cart start moving by the elastic cord.




Picture 3.4.3 starting experiment in "Sheet1" : If you release the cart at the point of (d), the cart will start moving.

3. When the cart stops moving, click [Stop Experiment] button in [Experiment] window and stop collecting data.

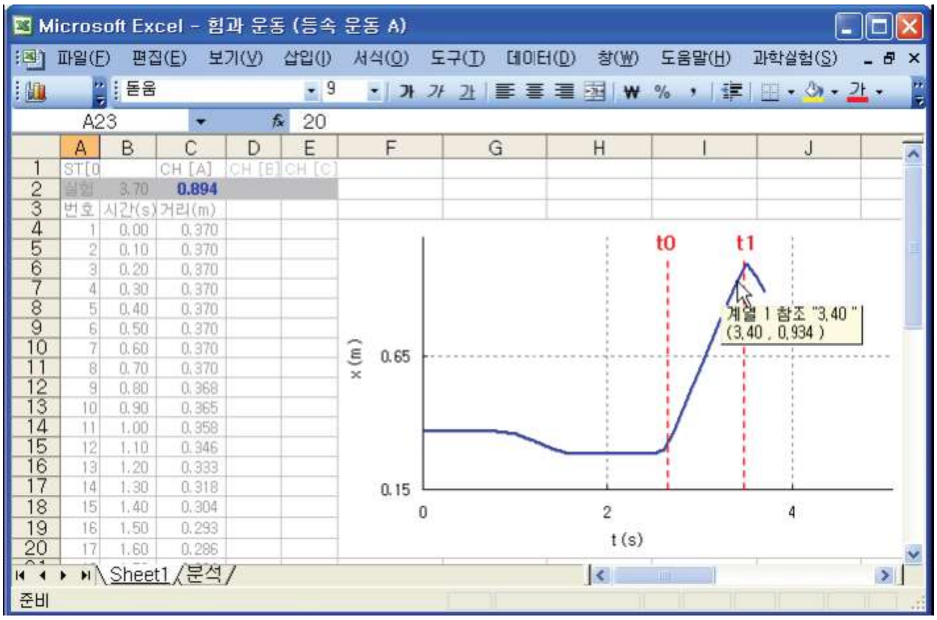
4. After finishing collecting data, analyze the experiment.

a. As in picture 3.4.4, in "Sheet1", read the time (x axis) value of the time section between t0 and t1[62], in which the cart does the uniform motion.

---

[61] This is the state that the cart stops because the cart is held after moving the cart by releasing the elastic cord.

[62] When you put the cursor onto the graph curve, the balloon message appears and it shows the (x, y) value of one point on the curve.

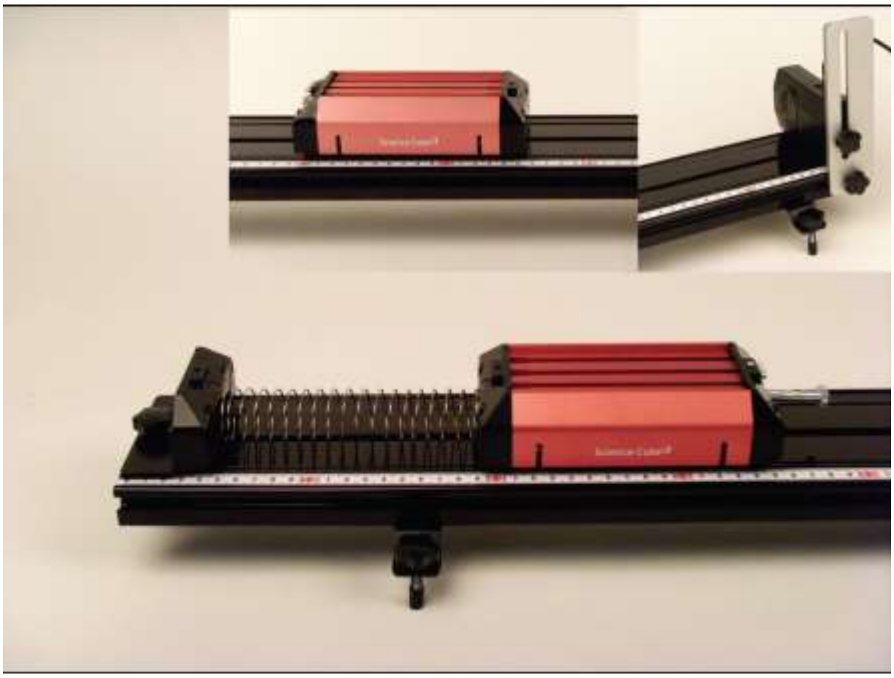



Picture 3.4.4 reading time section in “Sheet1”: In “Sheet1”, you should read the time section between t0 and t1 that will be analyzed.

b. As in picture 3.4.5, open “Analysis” sheet and input the value of time t0 in cell D1 and of time t1 in cell D2, and click [Experiment Analysis] to analyze the experiment data.


Picture 3.4.5 analyzing experimental data in “Analysis” sheet

## Experiment Process B: Cart's Motion by Spring



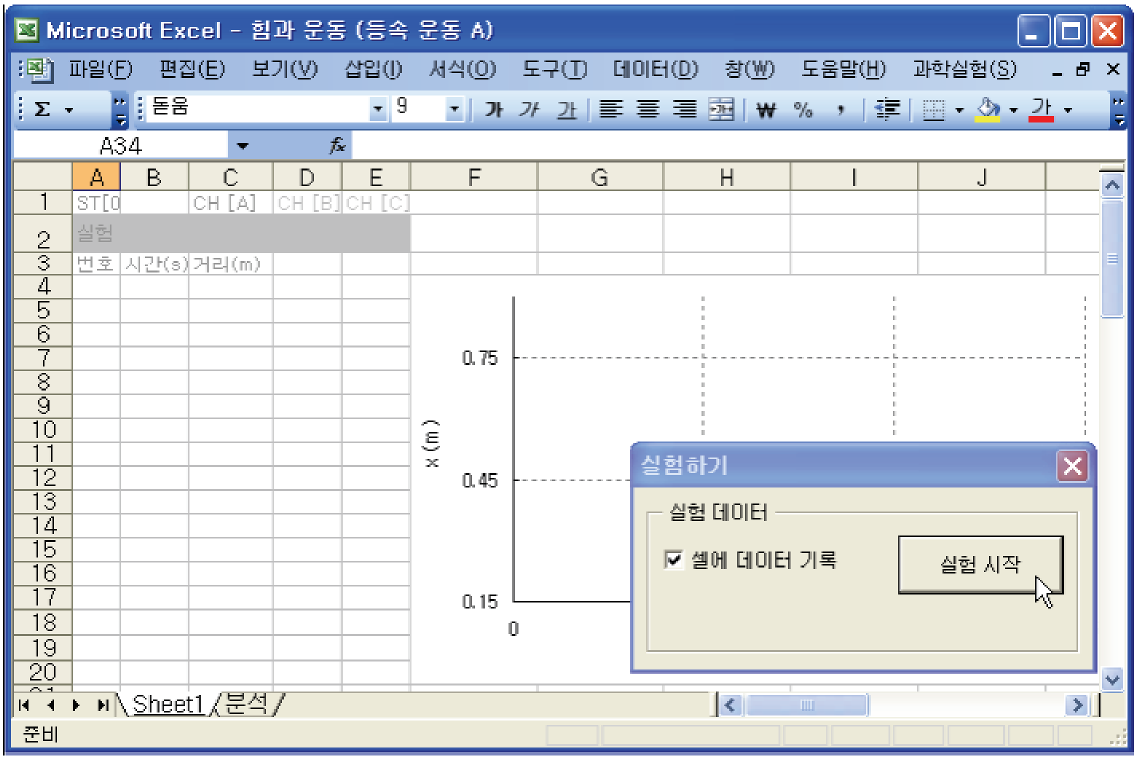
Picture 3.4.6 cart's motion by spring

1. As in picture 3.4.6, set up the spring at the left end of the track and set up the motion sensor at the right end.

2. Connect the motion sensor and the computer, open "Force and Motion (Uniform Motion A).xls" file and execute the experiment.

   a. Open "Sheet1" in Excel and open [Science Cube]-[Experiment Setting] window in worksheet menu to set up the estimating interval as 0.1 second, and the experiment time as 15 seconds.

   b. As in picture 3.4.2, open [Science Cube]-[Experiment] window in worksheet menu of "Sheet1" and click [Start Experiment] button.



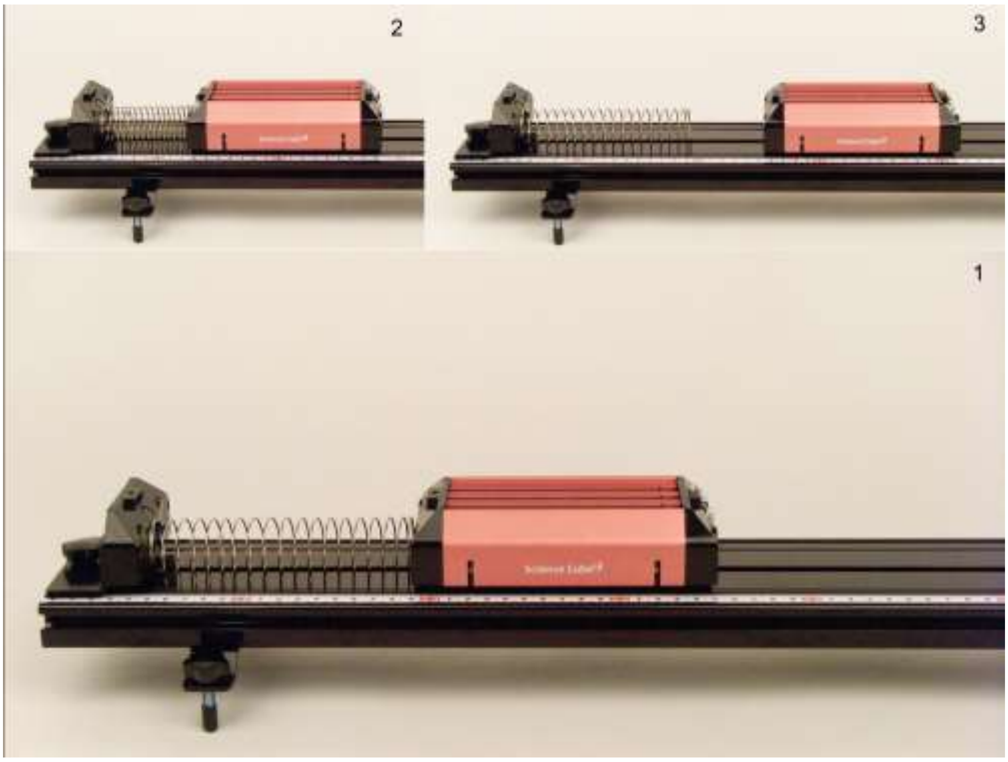

Picture 3.4.7 experiment with opening “Sheet1” of Excel workbook: If you click [Start Experiment] button, data will be collected in the sheet.

c. If you click [Start Experiment] button, as the graph section (a)-(b) in picture 3.4.9, the experiment data will be collected within the sheet of workbook.

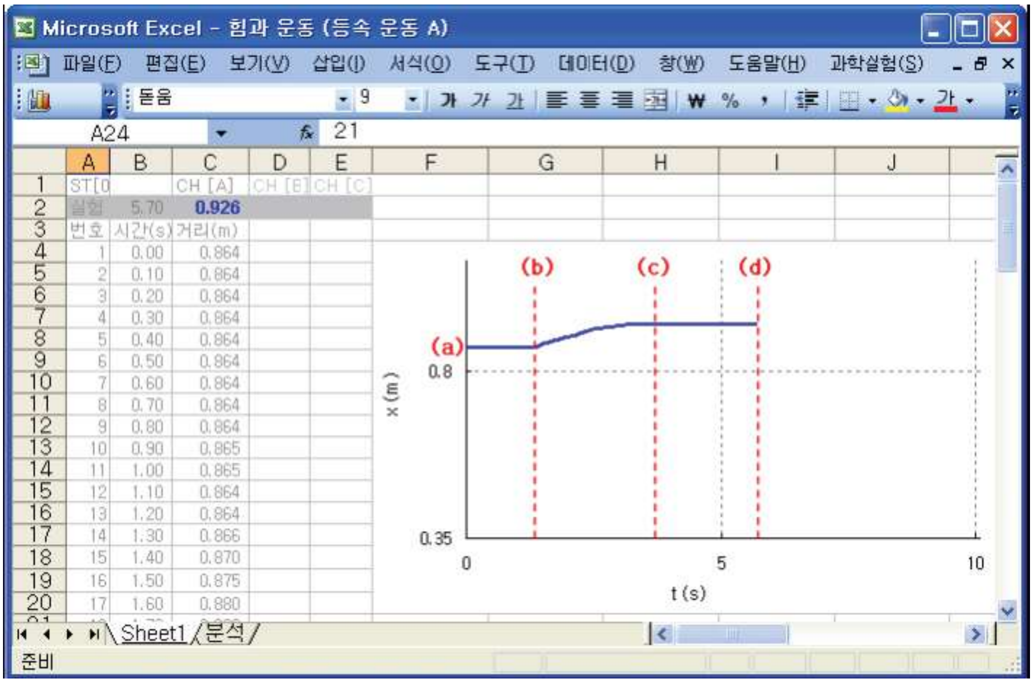
d. If you move the cart slowly toward the motion sensor, the data will be collected as the graph section (b)-(c) in picture 3.4.9. Section (c)-(d) is the state of staying after moving the cart.

e. Release the cart slightly at the point of (d) in picture 3.4.9[63] and make the cart start moving by the spring.

[63] The shape of $x-t$ graph is different from the cart’s motion by the elastic cord in picture 3.4.3. It’s because the motions sensor’s direction is different. In the cart’s motion according to the spring as in picture 3.4.8, the cart moves toward the motion sensor, but in the motion according to the elastic cord, the cart does the motions that gets far form the motion sensor.




Picture 3.4.8 making cart's motion with the spring[64]



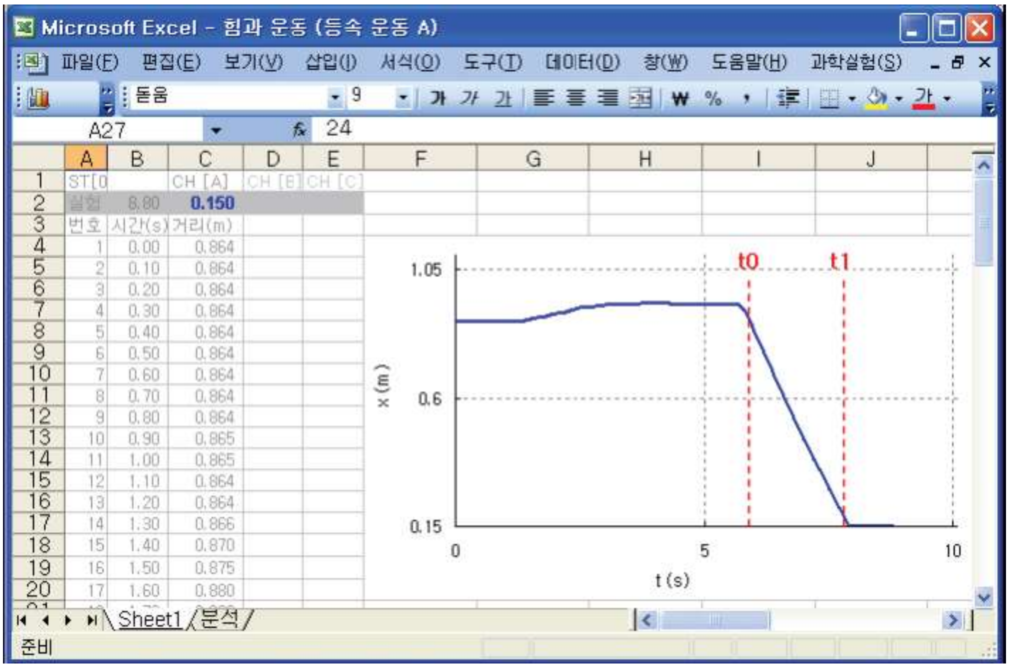

Picture 3.4.9 starting experiment in "Sheet1" : If you release the cart at the point of (d), the cart will start moving.

[64] Release the cart after pushing it toward the spring so that the spring is compressed.

3. When the cart stops moving, click [Stop Experiment] button in [Experiment] window and stop collecting data.

4. After finishing collecting data, analyze the experiment.

a. As in picture 3.4.10, in “Sheet1”, read the time (x axis) value of the time section between t0 and t1[65], in which the cart does the uniform motion.




Picture 3.4.10 reading time section in “Sheet1”: In “Sheet1”, you should read the time section between t0 and t1 that will be analyzed.

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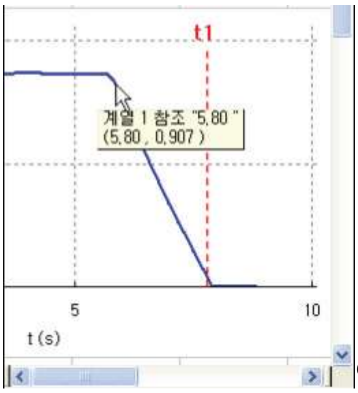
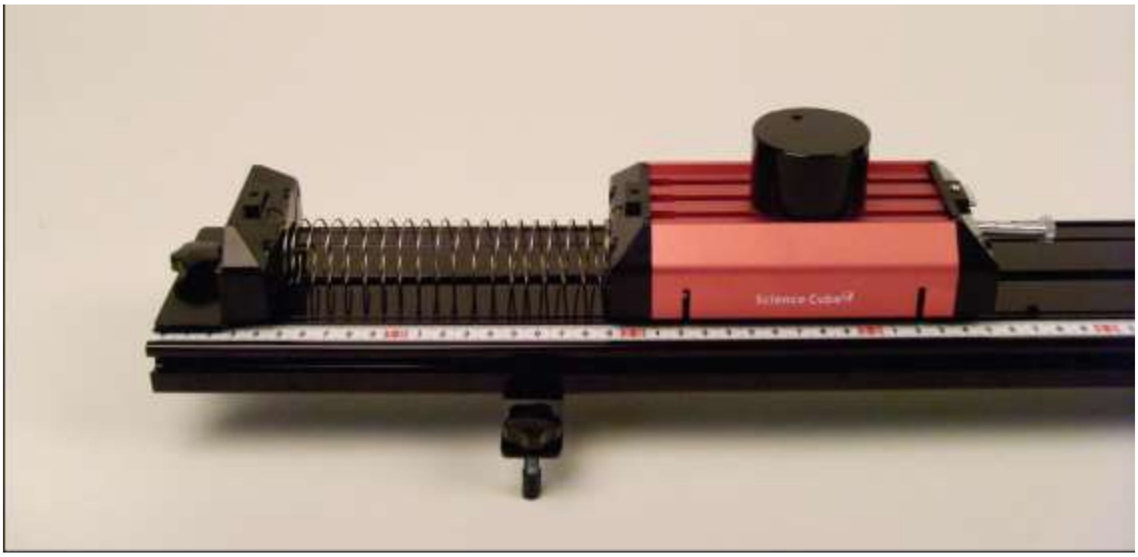


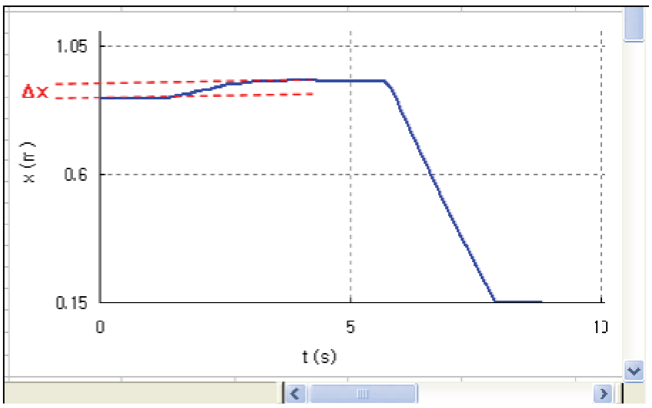

65 As the picture above, when you put the cursor onto the graph curve, the balloon message appears and it shows the (x, y) value of one point on the curve. At this moment, the data series values of the curve are named automatically just as “Series 1”. In the picture (x,y)=(5.80, 0.907).

b. Open “Analysis” sheet and input the value of time t0 in cell D1 and of time t1 in cell D2, and click [Experiment Analysis] to analyze the experiment data.

c. Repeat the experiment by changing the spring’s length of compression[66] differently. Also, with constant compressed length, change the cart’s mass as in picture 3.4.11 and repeat the experiment.



Picture 3.4.11 changing the cart’s mass: You can make the total mass[67] bigger by putting a pendulum of mass m on the cart of mass M.




[66] As the picture above, when you read $\Delta x$ in s-t graph, it is the spring’s compressed length. Put the cursor on the curve and read the scale of y axis x(m).

[67] If you input the value of the additional mass m in “Analysis” sheet, you can analyze the motion result for the cart’s total mass.

## Experiment Explanation:

1. Record the analyzed values of the cart's time t(s), location x(m), and velocity v(m/s) in the table below.

| Time t(s) | Location x(m) | Velocity v(m/s) |
|---|---|---|
| | | |

Table 3.4.1 experiment result : analyzed value of the cart's location and velocity

2. Read cell E4 or E5[68] in "Analysis" sheet of Excel workbook in which the experiment is done. What is the value of the cart's velocity? Explain it with the graph's gradient.

The cart's velocity (AVG) ____________________m/s
The cart's velocity (SLOPE) ____________________m/s

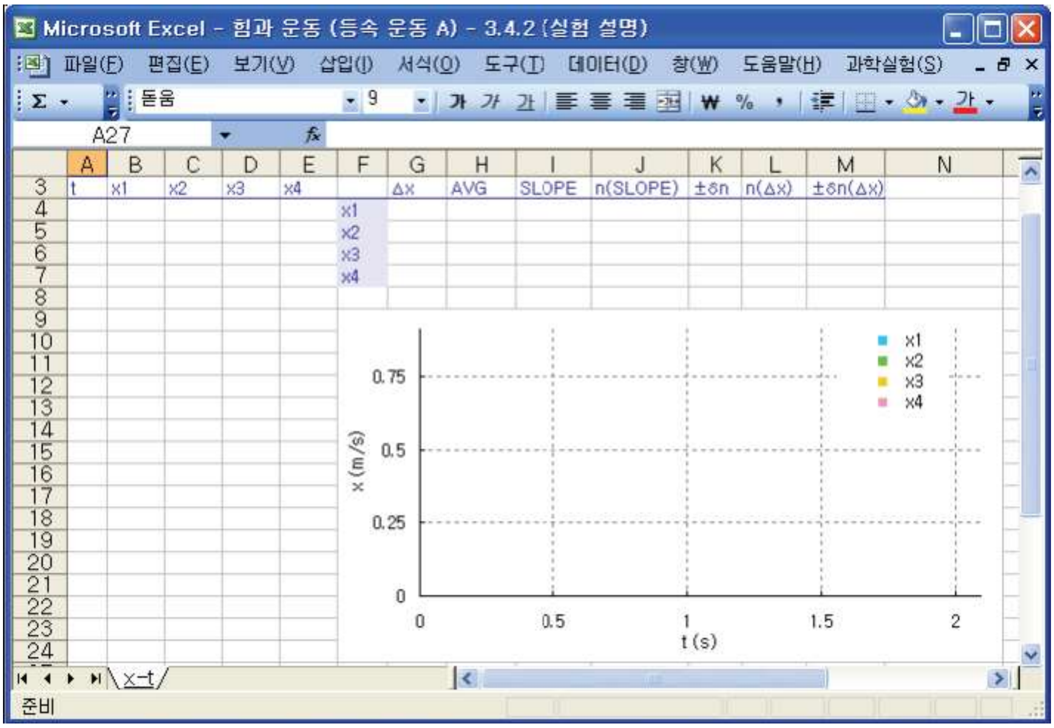
3. How is the cart's velocity different when the elastic cord is pulled longer or shorter? Explain the fact learned from the experiment result.

4. How is the cart's velocity different according to the spring's compressed length?

a. As in picture 3.3.12, draw $x-t$ graphs according to the compressed length x1, x2, x3, and x4 in one chart.

b. In table 3.3.2, interpret the experiment result about the ratio of length n($\Delta x$) and the ratio of velocity n(SLOPE) within the error range.

[68] E4 is the average value of the velocity between the time section t0 and t1, and E5 is the gradient of $x-t$ graph.




Picture 3.4.12 comparing and analyzing $x-t$ graph according to the spring's length

| | Δx | AVG | SLOPE | n (SLOPE) | ±δn (SLOPE) | n (Δx) | ±δn (Δx) |
|---|---|---|---|---|---|---|---|
| x1 | | | | 1 | - | 1 | - |
| x2 | | | | | | | |
| x3 | | | | | | | |
| x4 | | | | | | | |

Table 3.4.2 result of analysis according to the spring's length[69]

4. When the spring's compressed length is constant, how is the cart's velocity different according to the cart's mass?

[69] The error is as follows.

$$\delta n(v)=n(v)\sqrt{\left(\frac{\delta v}{v_1}\right)^2+\left(\frac{\delta v}{v_2}\right)^2},\quad \delta n(\Delta x)=n(\Delta x)\sqrt{\left(\frac{\delta\Delta x}{\Delta x_1}\right)^2+\left(\frac{\delta\Delta x}{\Delta x_2}\right)^2}.$$

When the error of the motion sensor is known as $\delta x=\pm 0.002$, $\delta v\approx 0.028, \delta\Delta x\approx 0.003$.

### 3.4.3 Experiment B: Motion That Changes the Direction

---

Experiment Prediction:

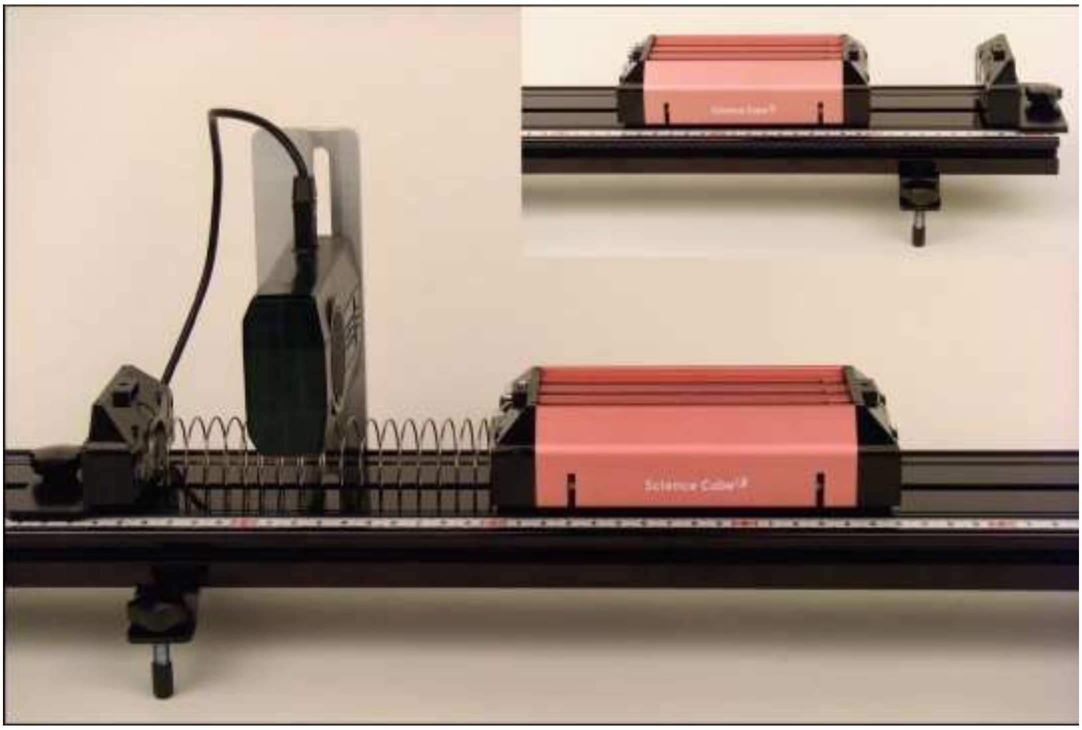
1. When the cart is repulsed by the magnet bumper and the moving direction becomes the opposite, how will the velocity before and after the repulse change?

a. What is the ratio of the velocity before and after the repulse?

b. When the cart's direction changes, predict the case that the cart's momentum is almost preserved and the case that it is not. What is the role of the magnet bumper on the change of the cart's momentum?

---

Experiment Process: Motion That Changes Direction and Preservation of Momentum



Picture 3.4.13 motion in which the cart's direction is changed

1. As in picture 3.4.13, set up the spring at the left end of the track and set up the motion sensor above it[70].

2. Put neodymium magnets[71] in the cart's bumper and the stopping bumper. The same poles of magnets should face each other so that the cart and the stopping bumper can push each other away.

3. Connect the motion sensor and the computer, open "Force and Motion(Uniform Motion B).xls" file and execute the experiment.

   a. Open "Sheet1" in Excel, open [Science Cube]-[Experiment Setting] window in worksheet menu and set up the estimating interval as 0.05 second and the experiment time as 2.5 seconds.

   b. As in picture 3.4.14, choose [Science Cube]=[Experiment] in worksheet menu of "Sheet1" and open [Experiment] window[72].

   c. Move the cart slowly toward the motion sensor, compress the spring as much as the constant displacement $\Delta x$[73] and release it.

   d. When the cart is pushed away by the spring, starts moving toward the right side of the track and comes within the range of 0.35m before the stopping bumper[74], click [Start Experiment] button.

---

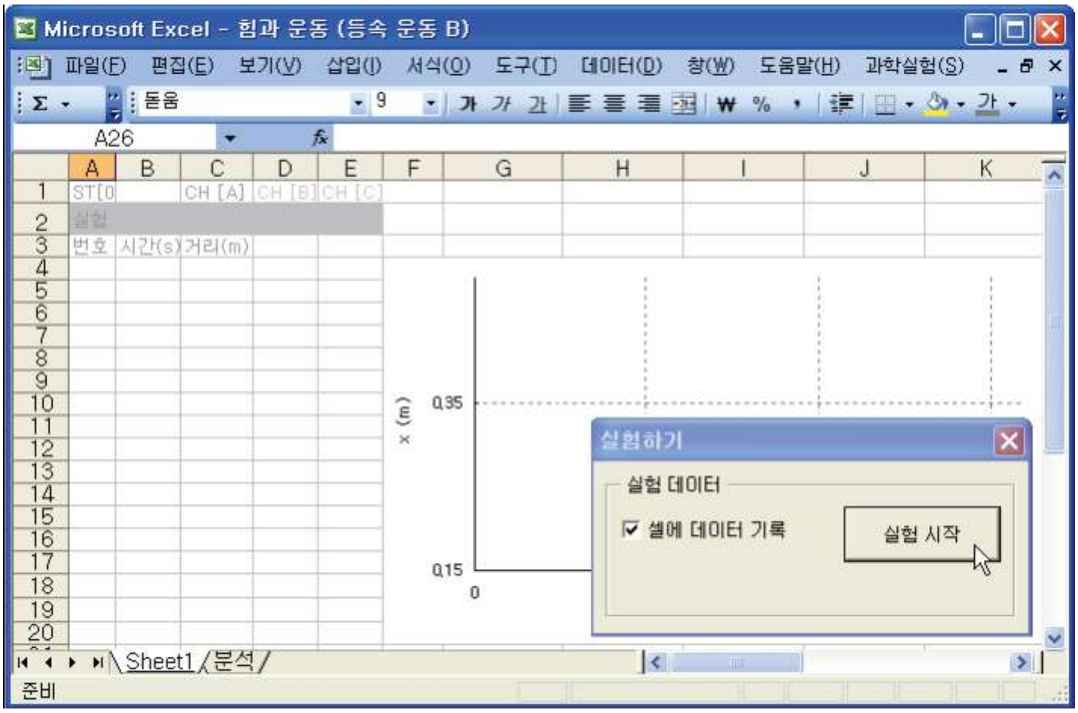
[70] Leave some space so that the spring does not touch the motion sensor.

[71] 

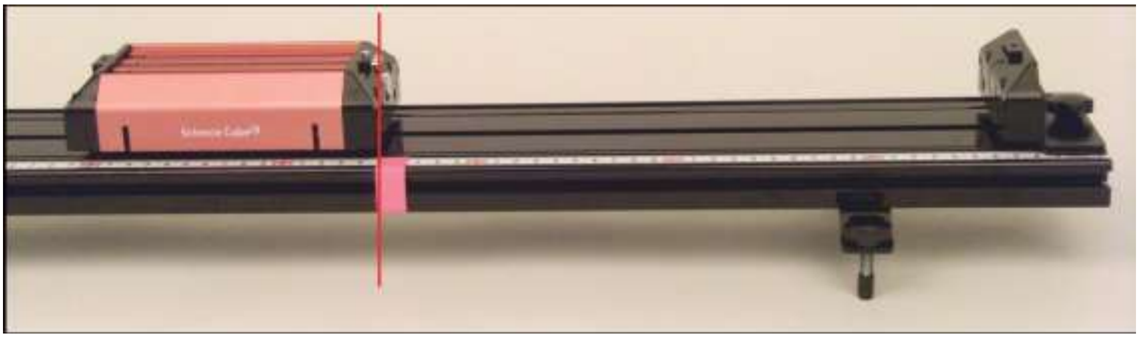
[72] Just open [Experiment] window and do not click [Start Experiment] button.

[73] You can do the experiment by changing the compressed displacement of the spring.

[74] This is the location that starts collecting data, so this distance can be changed according to the experiment situations.




Picture 3.4.14 experiment in "Sheet1": If you click [Start Experiment] button, data is collected within the sheet.



Picture 3.4.15 clicking [Start Experiment] button when the cart is in a certain position: In the process of d and f, start or stop the experiment when the cart enters or goes out the position indicated on the track.

e. When the cart is repulsed by the stopping bumper and moves leftward again, click [Start Experiment] button[75] and stop the experiment if it is diverted more than 0.35m from the bumper.

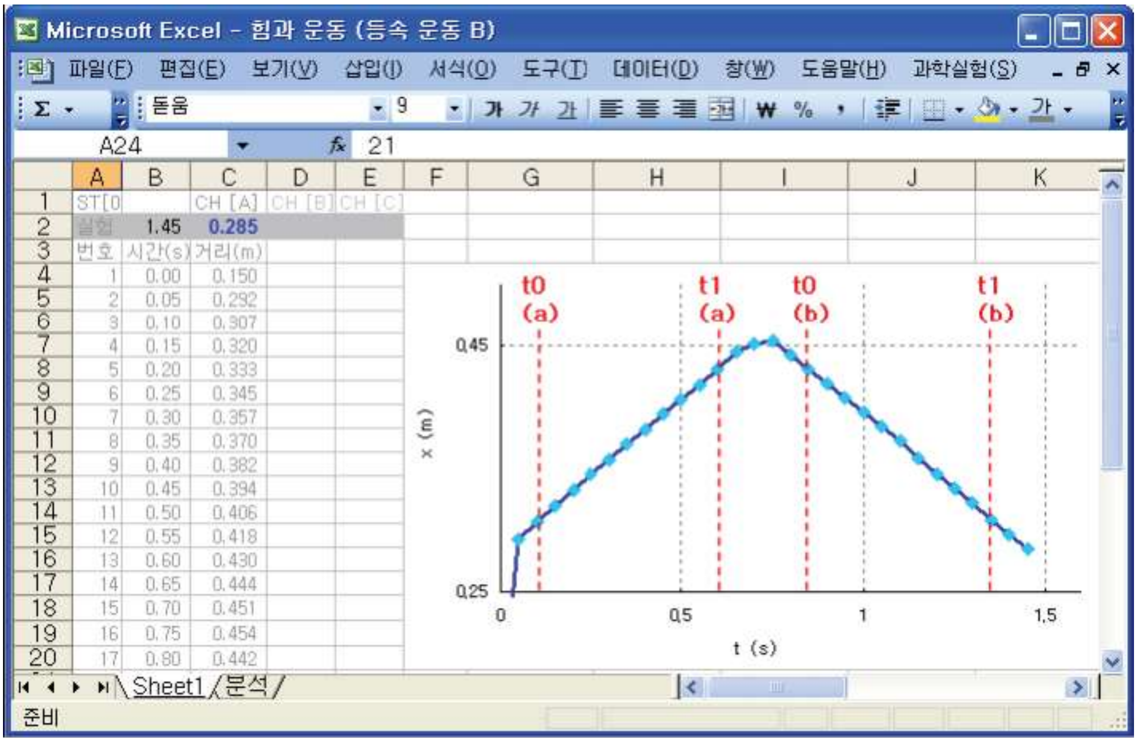
---

[75] If you don't click [Start Experiment] button, the data collecting stops after the experiment time set up in [Experiment Setting] is over.

4. After finishing data collecting, analyze the experiment.

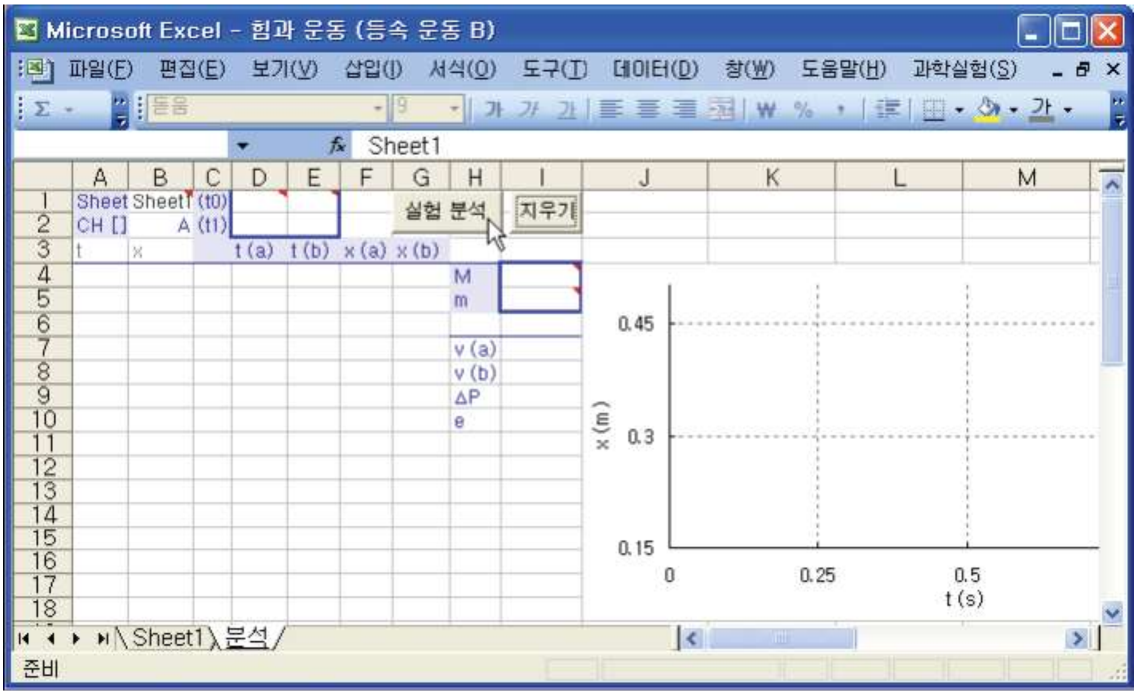
a. As in picture 3.4.16, in “Sheet1”, read the t0 and t1 time value(x axis) of section (a) in which the cart is not yet repulsed by the bumper and of section (b) i which the cart is repulsed.

b. As in picture 3.4.17, open “Analysis” sheet and input the time value of t0 and t1 of section (a) in cell D1 and D2. And input the time value of t0 and t1 of section (b) in cell E1 and E2. And input the cart’s mass in cell I4. Then click [Experiment Analysis] button and analyze the experiment data.




Picture 3.4.16 reading time section that will be analyzed in “Sheet1”: You should read the value of the time section between t0 and t1[76] in “Sheet1”.

[76] The experiment result will be changed according to the values of t0 and t1. It’s because the data of the analyzing section changes. For example, in case of the trend analysis such as the linear gradient, the reliability of the analysis can be understood by indicating the value of R- square. If the theoretical prediction is done before the experiment, you can choose the range by changing the value of t0 and t1 to get the experiment value which is nearest to the theoretical value.




Picture 3.4.17 analyzing the velocity of section (a), before the cart's direction is changed, and of section (b), after it is changed[77]

---

**Experiment Explanation:**

1. Read cell 17 and cell 18 in "Analysis" sheet of "Force and Motion (Uniform Motion B).xls" file. What is the velocity of section (a), in which the cart's direction does not change, and of section (b), in which the cart's direction changes?

The velocity of section (a) ________________m/s

The velocity of section (b) ________________m/s

---

[77] You can calculate the change of momentum $\Delta P$ in cell I9 if you input the cart's mass in cell I4 and the additional mass in cell I5. cell I10 is the ratio of the velocities before and after the cart is repulsed.

2. Calculate the cart's momentum and see if the momentum is preserved or not. And explain the change of momentum.

a. Calculate the cart's momentum in section (a), $P_a$ , and the cart's momentum in section (b), $P_b$ .

Section (a)'s momentum $P_a$ ____________________kgm/s

Section (b)'s momentum $P_b$ ____________________kgm/s

b. Read cell I9 in "Analysis" cell. This value is the change of the cart's momentum between section (a) and section (b).

The change of momentum $\Delta P$ ____________________kgm/s

c. Explain the cart's change of momentum $\Delta P$ . Without the magnet bumper[78], how will this value change? Explain the role of magnet bumper.

______________________________________________________________

______________________________________________________________

______________________________________________________________

[78] Execute experiments using various buffer materials such as paper or sponge. Also, use solid materials and do the experiment.

### 3.4.4 Experiment C: Two Carts' Motion

---

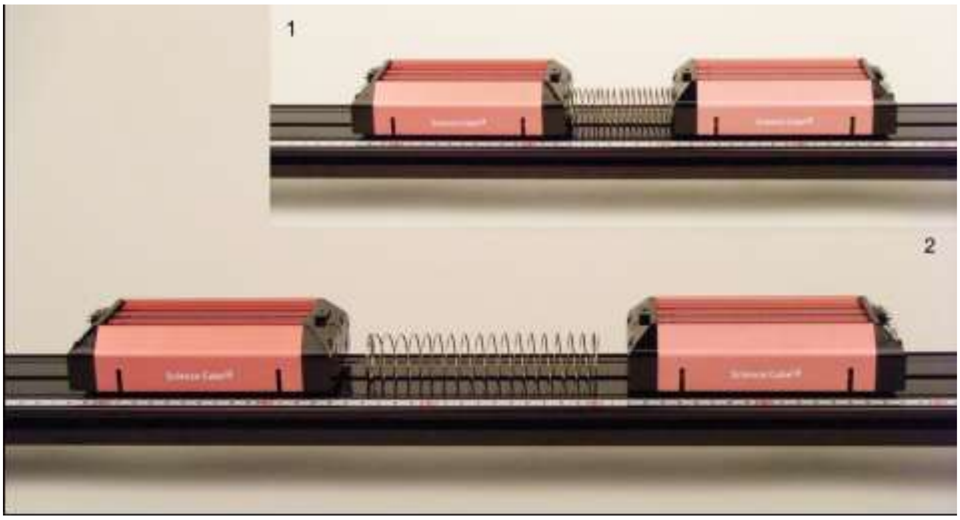
**Experiment Prediction:**

1. When the two carts that are pushed away by the spring have same masses, how will the ratio of two carts' repulsed velocities become?

2. What can be explained with the ratio of the carts' velocities and of the carts' masses?

---

**Experiment Process: Two Carts' Velocities That Are Repulsed and Moving**

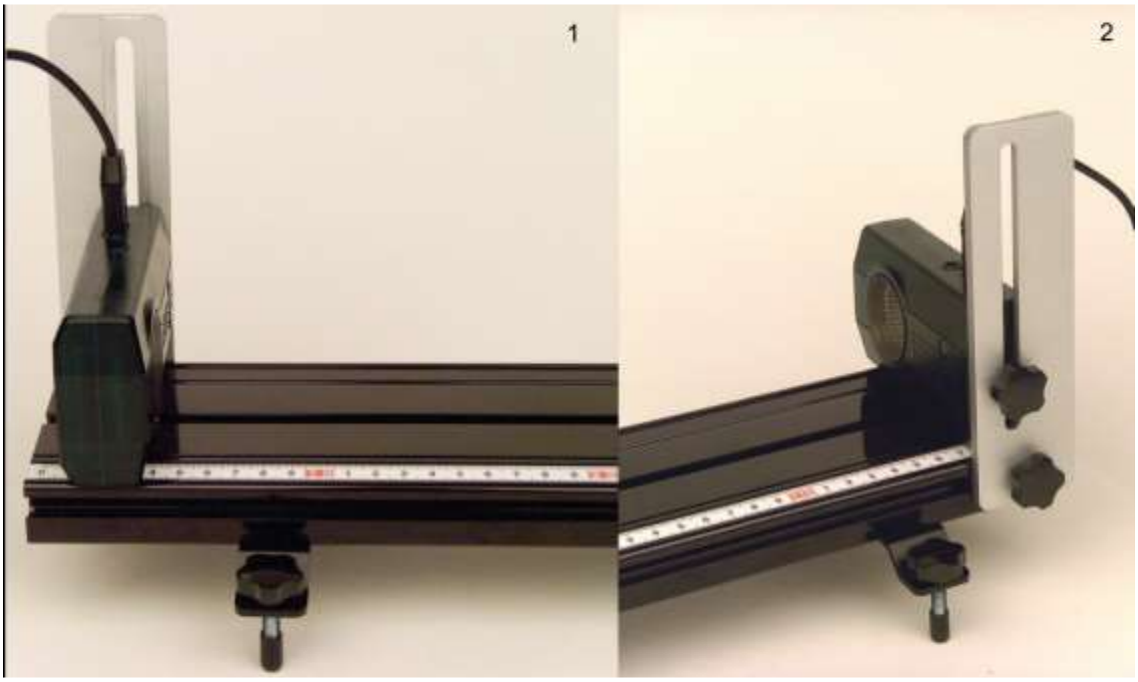



Picture 3.4.18 motion of two carts which are repulsed by the spring and get far away from each other: 1- the spring is compressed. 2- two carts are released and they are in the motion of being far away form each other.

1. As in picture 3.4.19, set up motion sensor A and B each at both ends of the track[79].

---

[79] Motion sensor A at the left end senses cart A on the left and motion sensor B at the right end senses cart B on the right.




Picture 3.4.19 setting up motion sensors at both ends of the track : You should connect motion sensor A on the left end and motion sensor B on the right end so that the data should be collected in channel A and B.

2. Connect each motion sensors to channel A and B and open "Force and Motion(Uniform Motion C).xls" file[80] to do the experiment.

a. Open "Sheet1" of Excel. Open [Science Cube]-[Experiment Setting] window in worksheet menu and set up the estimating interval as 0.1 second and the experiment time as 15 seconds[81].

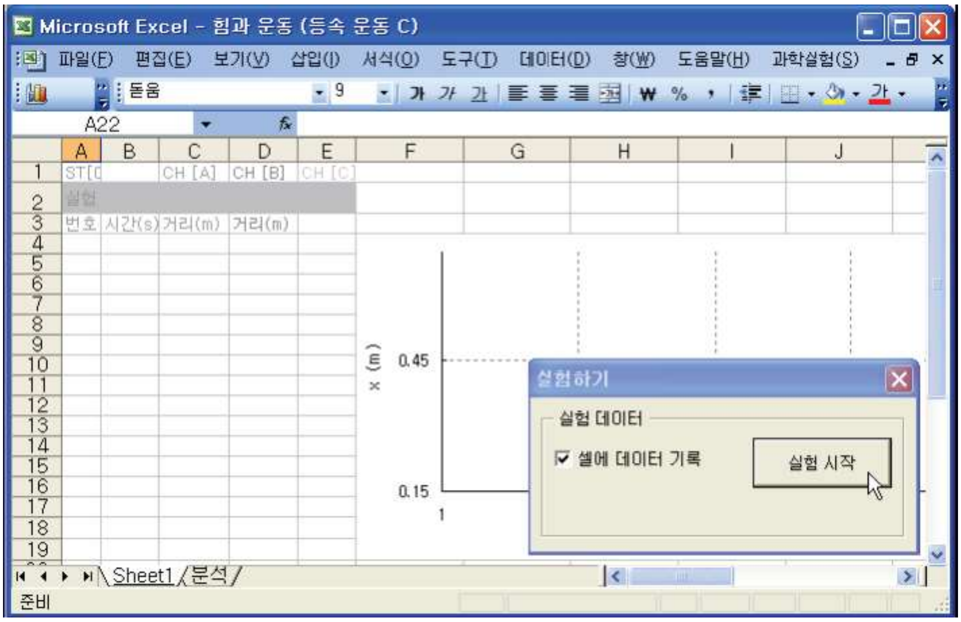
b. As in picture 3.4.20, open [Science Cube]-[Experiment] window in worksheet menu and click [Start Experiment] button.

c. When you click [Start Experiment] button, the experiment data is collected within the sheet of workbook just as the graph section (a)-(b) in picture 3.4.21.

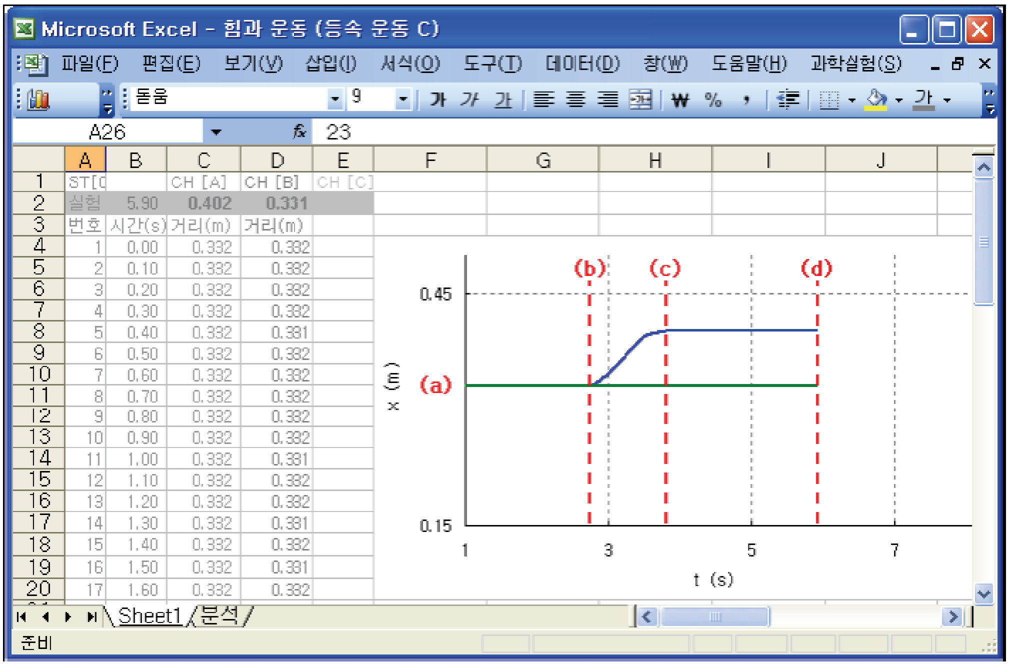
---

80 Dispersed chart has been made already in "Force and Motion (Uniform Motion C).xls" file so that $x-t$ graph can be drawn with the cart's location data collected from channel A and channel B.

81 The experiment time can be changed according to the situations. Do the pre-experiment so that the data collecting can be finished within the experiment time.




Picture 3.4.20 experiment with opening “Sheet1” : If you click [Start Experiment] button, data will be collected within the sheet.

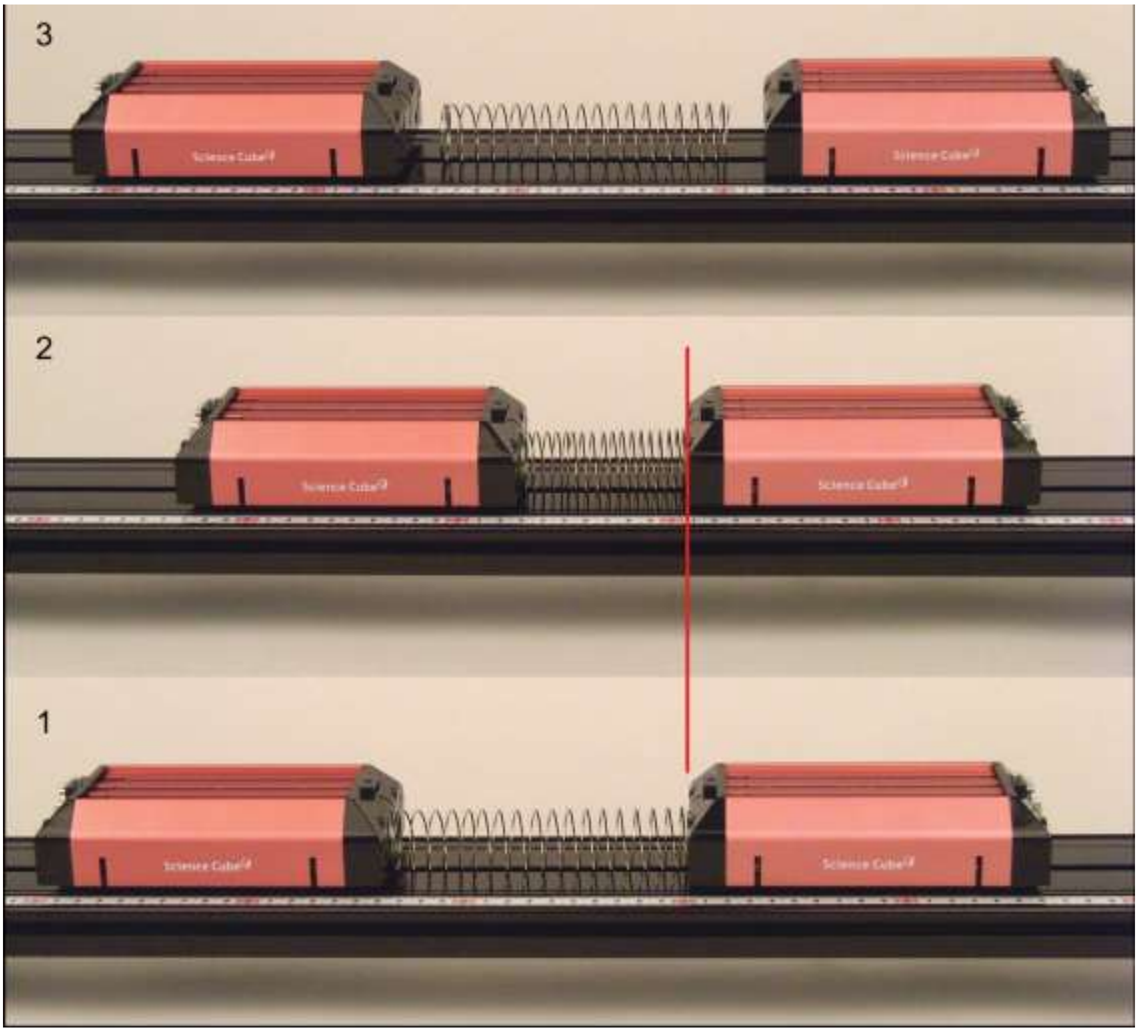



Picture 3.4.21 starting experiment in “Sheet1” : When you release the cart at the point of (d)[82], the cart will begin to move.

[82] If you do the experiment as a pair, you can click [Start Experiment] button in (c)-(d) section and execute the experiment.

d. Hold the cart on the right tightly. Push the cart on the left toward the right one and compress the spring. The data is collected as the graph section (b)-(c) in picture 3.4.21. Section (c)-(d) is the state that the cart has been moved and stayed still.

e. When you release the cart slightly on the point of (d) in picture 3.4.21, the cart is pushed away by the spring's force[83] and starts moving. Picture 3.4.22 shows this process.



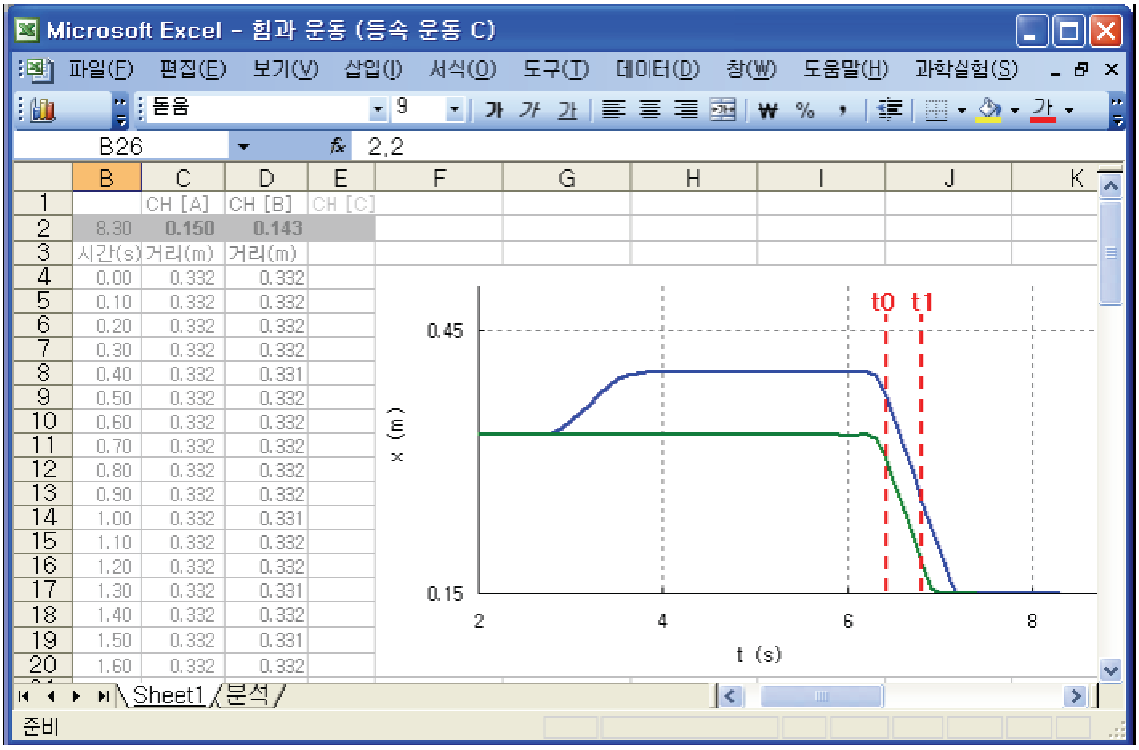

Picture 3.4.22 process that the two carts are repulsed and start to move according to the spring

[83] You can do the experiment and learn how the cart's velocity changes according to the spring's compressed length.

3. When the carts stop moving, click [Stop Experiment] button in [Experiment] window and stop collecting data.

4. After the data collecting is finished, analyze the experiment.

a. As in picture 3.4.23, in "Sheet1", read the time value of section t0 and t1(x axis) in which the cart is in the uniform motion.



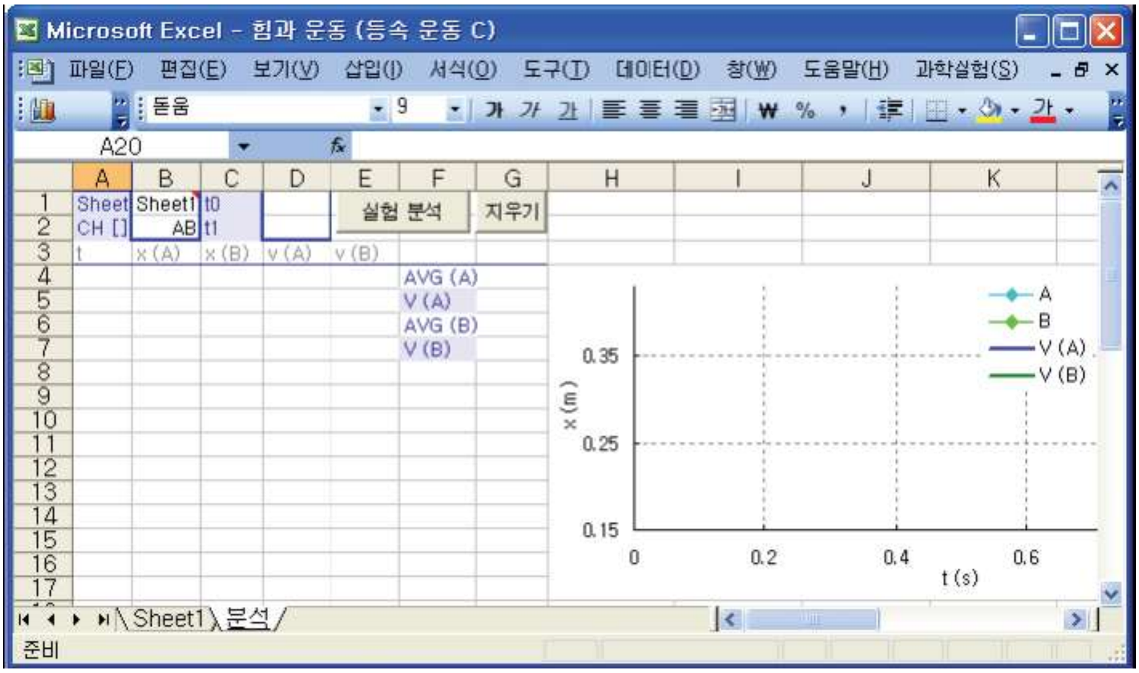

Picture 3.4.23 reading time section that will be analyzed in the sheet : In "Sheet1", you should read the value of time section t0 and t1 that will be analyzed.

b. Open "Analysis" sheet and input the value of time t0 in cell D1, and of time t1 in cell D2. If you click [Experiment Analysis] button, the values that analyze two carts' velocities[84] are recorded in cell G5 and G7 each.

c. Change two cart's masses and execute the experiment.

---

[84] AVG (A) and AVG (B), the average values of average velocity during the time interval $\Delta t$, are recorded in cell G4 and G6. Two carts' velocities, that is, the values of G5 and G7 correspond to the gradient of $x-t$ graph.




Picture 3.4.24 analyzing in “Analysis” sheet : If you click [Experiment Analysis] button, the velocities of channel A and channel B are analyzed.

## Experiment Explanation:

1. Record the analyzed values of two carts’ time t, locations x(A) and x(B), and velocities v(A) and v(B) in the table below.

| t | x (A) | x (B) | v (A) | v (B) |
|---|---|---|---|---|

Table 3.4.3 analysis result of two carts’ motion experiment

2. Read Cell G5 and G6 in “Analysis” sheet. How are the two carts’ velocities? Express the result including the experiment error[85].

Cart (A)’s velocity ____________________ ± ______________m/s

Cart (B)’s velocity ____________________ ± ______________m/s

3. When two carts’ masses are different from each other, analyze the result of velocity within the experiment error range.

### 3.4.5 Experiment Questions

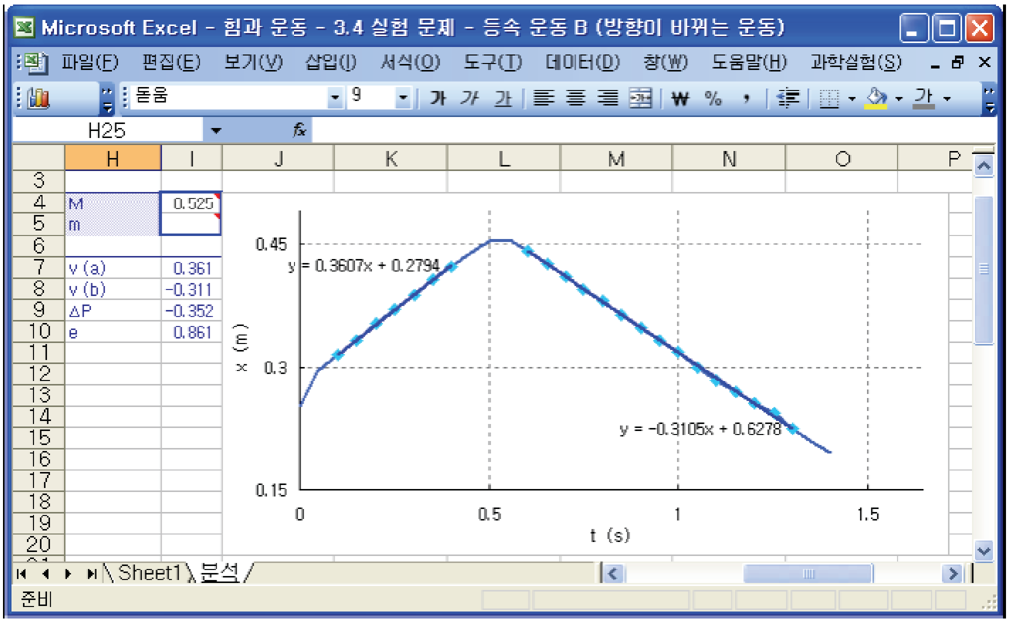
1. Table 3.4.4 is the result of velocity when the spring is compressed as much as $\Delta x$ and pushes the cart. Within the error range, analyze and explain whether the ratio of spring’s compressed length is same as the ratio of velocity. The error of the motion sensor is $\delta = \pm 0.0002$.

| | Δx | v |
|---|---|---|
| x1 | 0.057 | -0.308 |
| x2 | 0.064 | -0.365 |
| x3 | 0.079 | -0.461 |

Table 3.4.4 result of experiment analysis according to the spring’s compressed length

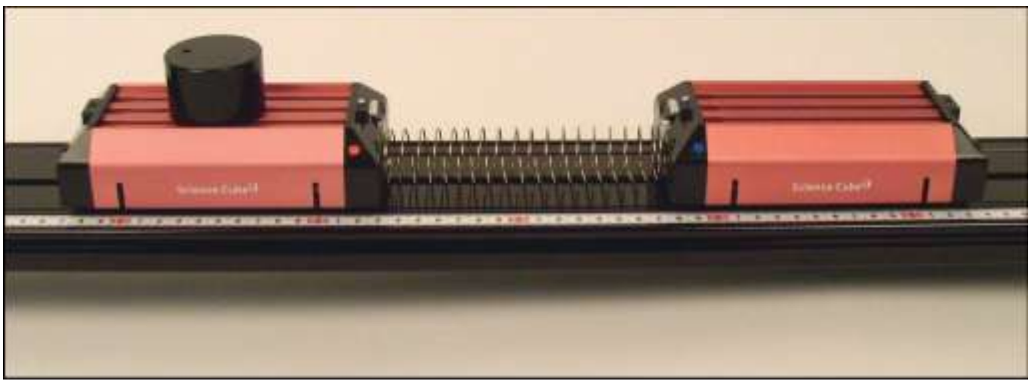
85 In this experiment, the error of the motion sensor is $\delta x = \pm 0.0002$.

2. Picture 3.4.25 is the result of experiment in which a cart of 0.525 kg is repulsed by the stopping bumper. The velocity before the repulsion is 0.361m/s and the velocity after the repulsion is -0.311m/s. Explain the change of the cart's momentum.



Picture 3.4.25 result graph of the cart's motion experiment in which the cart's direction changes oppositely

3. When the spring is compressed and released as in picture 3.4.26, the spring pushes away two carts. In the analysis result, two carts' velocities are -0.309m/s and -0.304m/s each. If a pendulum of 0.502kg is added to one cart, how will the ratio of velocity change?



Picture 3.4.26 motion in which the direction changes oppositely

# 3.5

# Experiment: Velocity, Acceleration and Energy

## 3.5.1 Experiment Outline

Using the first dimensional motion track and a cart, estimate the cart's time and location in the situation in which constant force is working such as the motion on a slope or the motion according to the pendulum's falling and in the situation in which the force changes regularly according to the velocity. Analyze the cart's velocity and acceleration using these experiment data and study about the force's action and energy.

### Goal

Learning the concept of velocity, acceleration and energy in the situation of uniformly accelerated motion and understanding the force's action and the object's motion.

### Required Equipments

| | | | |
|---|---|---|---|
| Electronic scale | 1 | Pendulum(0.5kg) | 1 |
| Motion sensor | 2 | Pendulum(50g) | 2 |
| Springs(0.142m, 22.895N/m) | 1 | Holding tape | 1 |
| Neodymium magnets | 4 | Cord(1.2m) | 1 |

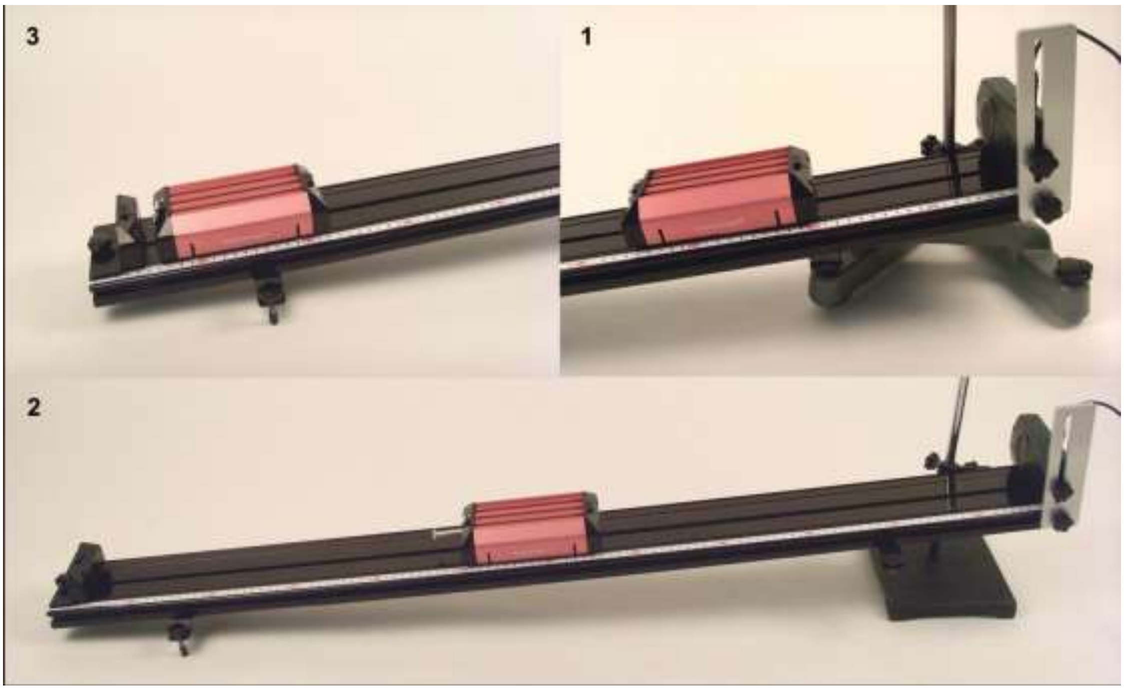
### 3.5.2 Experiment: Motion on the Slope

**Experiment Prediction:**

1. When the angle of the slope is constant, how is the cart's velocity that comes down the slope is different according to the slope's length?

2. How is the cart's acceleration different on the slop according to the angle? When the angle is $3 \pm 0.2^{\circ}$, calculate the cart's acceleration including the error.

**Experiment Process: Cart's Motion According to the Slope's Angle**




Picture 3.5.1 cart's motion on the slope

1. As in picture 3.5.1, combine the track with the lab stand[86] and set up the motion sensor at the right end.

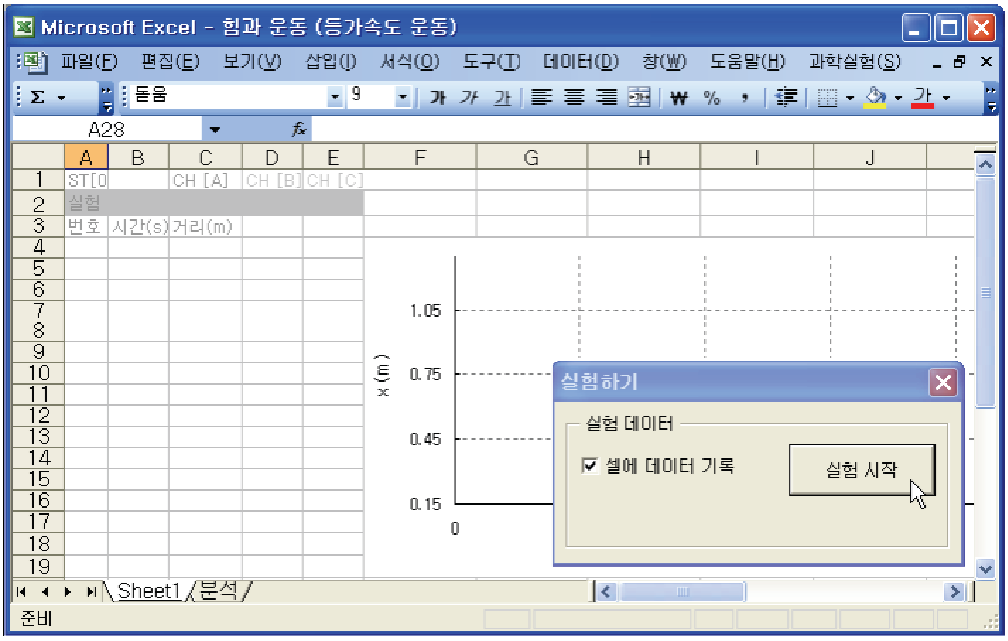
[86] You can change the slope's angle by adjusting the height.

2. Connect the motion sensor and the computer and estimate the slope's angle[87].

3. Open "Force and Motion(Uniformly Accelerated Motion).xls" file and execute the experiment.

a. Open "Sheet1" of Excel and open [Science Cube]-[Experiment Setting] window in the worksheet menu and set up the estimating interval as 0.1 second, and the experiment time as 5.0 seconds.

b. As in picture 3.5.2, open [Science Cube]-[Experiment] window in "Sheet1".




Picture 3.5.2 experiment with opening "Sheet1": If you click [Start Experiment] button, the data will be collected within the sheet.

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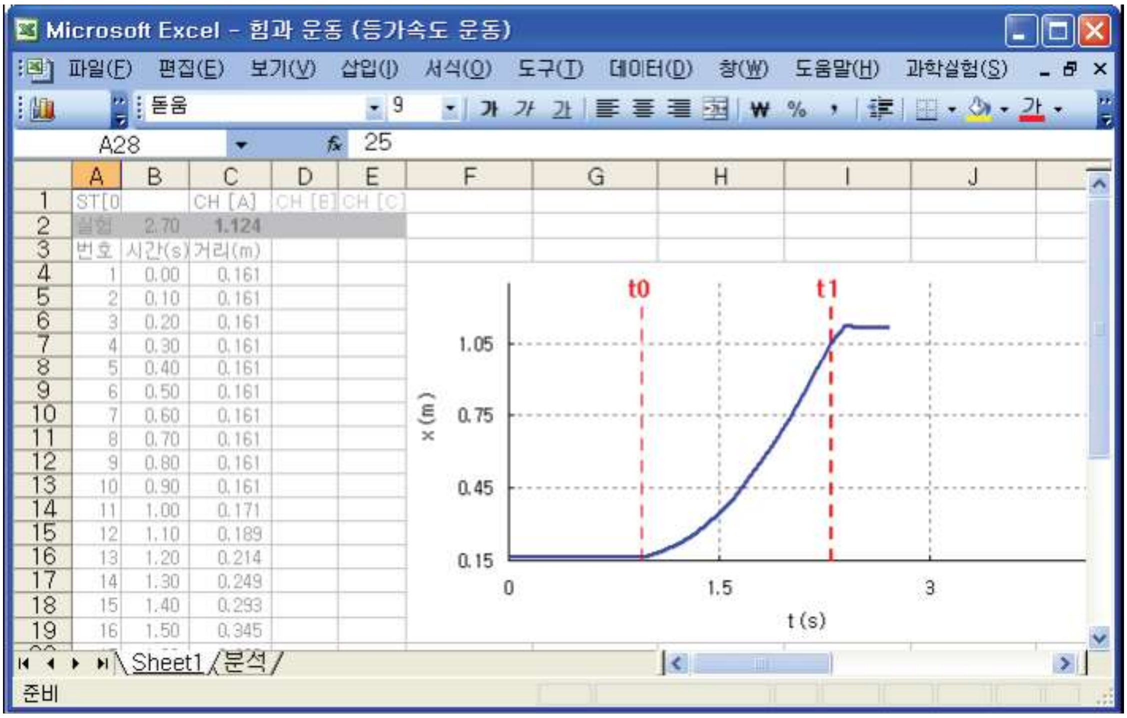
[87] When estimating the slope's angle, the estimating error according to the graduator's accuracy becomes the diffusion factor in the experiment analysis. When estimating the slope's angle, the uncertainty includes not only the systematic error(the error of the graduator itself) but also the parallax error(the error caused by reading angles with eyes not with the digital graduator).

c. Put the cart 0.2m ahead of the motion sensor[88] and hold it tight with one hand[89].

d. Click [Start Experiment] button in [Experiment] window. If you click it, the experiment data will be collected within “Sheet1”.

3. When the cart stops moving, click [Stop Experiment] button in [Experiment] window to stop collecting data and analyze the experiment.

a. As in picture 3.5.3, read the time value(x axis) of the time section t0 and t1 in “Sheet1”, in which the cart do the uniform motion.

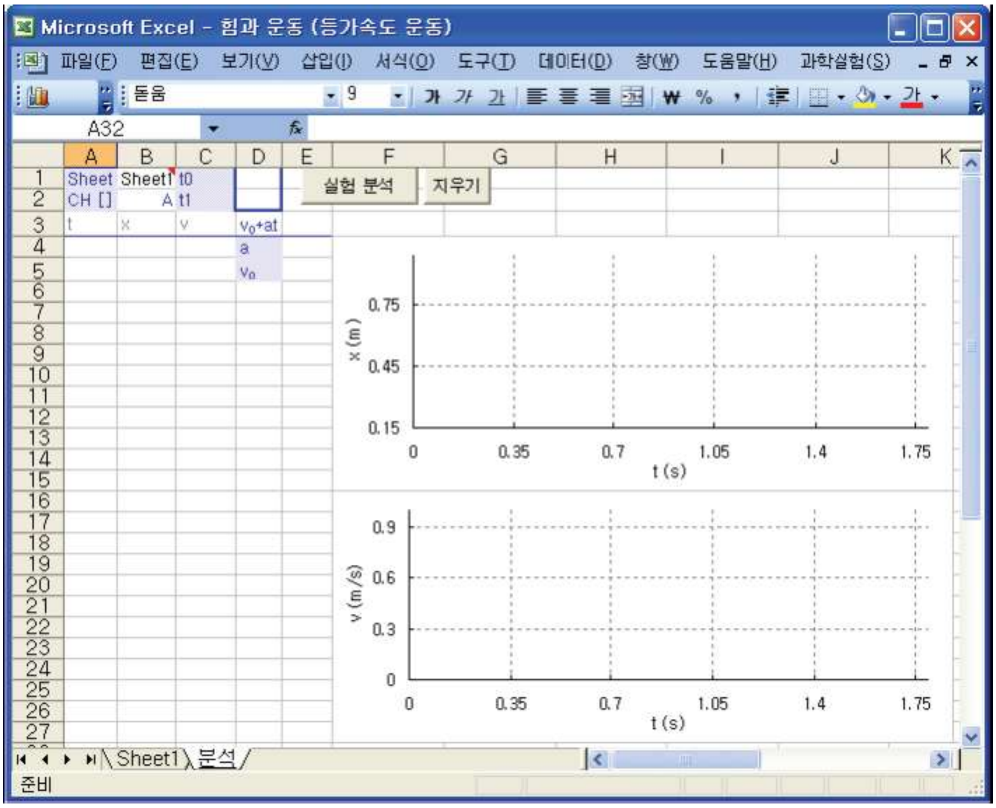



Picture 3.5.3 reading the time section that will be analyzed in “Sheet1” : You should read the time section t0 and t1 in “Sheet1”.

b. Open “Analysis” sheet and input the time t0 in cell D1 and the time t2 in cell D2. Click [Experiment Analysis] button and analyze the experiment data.

---

[88] Execute the experiment within the motion sensor’s sense range 0.15~6.0m.
[89] You can attach the cart’s side and the track’s side with the holding tape.




Picture 3.5.4 analyzing in "Analysis" sheet : If you click [Experiment Analysis] button, the velocity and the acceleration will be analyzed.

c. Change the slope's angle as $1°,3°,5°$ [90] and execute the experiment.

## Experiment Explanation:

1. Read the acceleration analyzed value in cell E4 and the initial velocity analyzed value $a$ in "Analysis" sheet.

The cart's acceleration ______________________m/s²

The cart's initial velocity ______________________m/s

[90] Considering the error's disperse according to the slope angle's error, execute the experiment by setting up the width of the angle bigger than $2°$.

2. Collect the experiment data when the slope's angles are $1.0 \pm 0.2°, 3.0 \pm 0.2°, 5.0 \pm 0.2°$ and draw $v-t$ graph.

3. Compare the acceleration a(T)'s theoretical value and experiment result within the experiment error range and calculate the cart's acceleration a and the gravity acceleration g. Do not consider the frictional resistance between the track and the cart and interpret the result within the experiment error range[91].

| | a | $g \pm \delta g*$ | $\pm \delta \sin\theta$ | a(T) | $\pm \delta g \sin\theta$ |
|---|---|---|---|---|---|
| 1.0±0.2° | | | | | |
| 3.0±0.2° | | | | | |
| 5.0±0.2° | | | | | |

Table 3.5.1 result of the cart's motion experiment according to the slopes' angle

4. Consider the friction between the track and the cart[92] and calculate the gravity acceleration g and express the result within the experiment error range. The error of the slope's angle is $\pm 0.2°$, and the distance estimating error of the motion sensor is $\pm 0.002m$.

The modulus of friction $\mu$ = ______________________ ________

The gravity acceleration g = ______________________ ________m/s²

[91] $\delta g*$ is the error about the acceleration of gravity and it can be calculated as follows.

$$\delta \sin\theta = \cos\theta \cdot d\theta, \quad \delta g = g\sqrt{\left(\frac{\delta a}{a}\right)^2 + \left(\frac{\delta \sin\theta}{\sin\theta}\right)^2}$$

[92] When there is the friction between the track and the cart, the force acted upon the cart can be expressed as below.

$$m(g \sin\theta - \mu g \cos\theta)$$

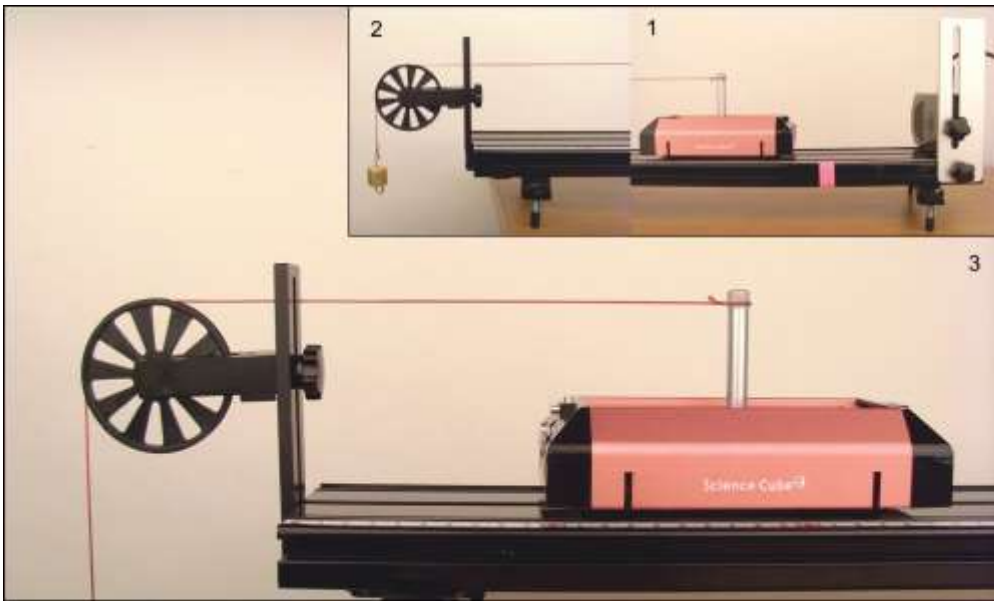
### 3.5.3 Experiment: Motion according a Pendulum's Falling

**Experiment Prediction:**

1. As in picture 3.5.5, when the cart gets the acceleration according to the pendulum's falling, how is it different according to the pendulum's mass or the cart's mass?

2. Draw graphs of the cart's potential energy according to time t, V(t), the kinetic energy T(t) and the total energy E(t).

**Experiment Process: Cart's Motion According to the Pendulum's Falling**




Picture 3.5.5 cart's motion according to the pendulum's falling

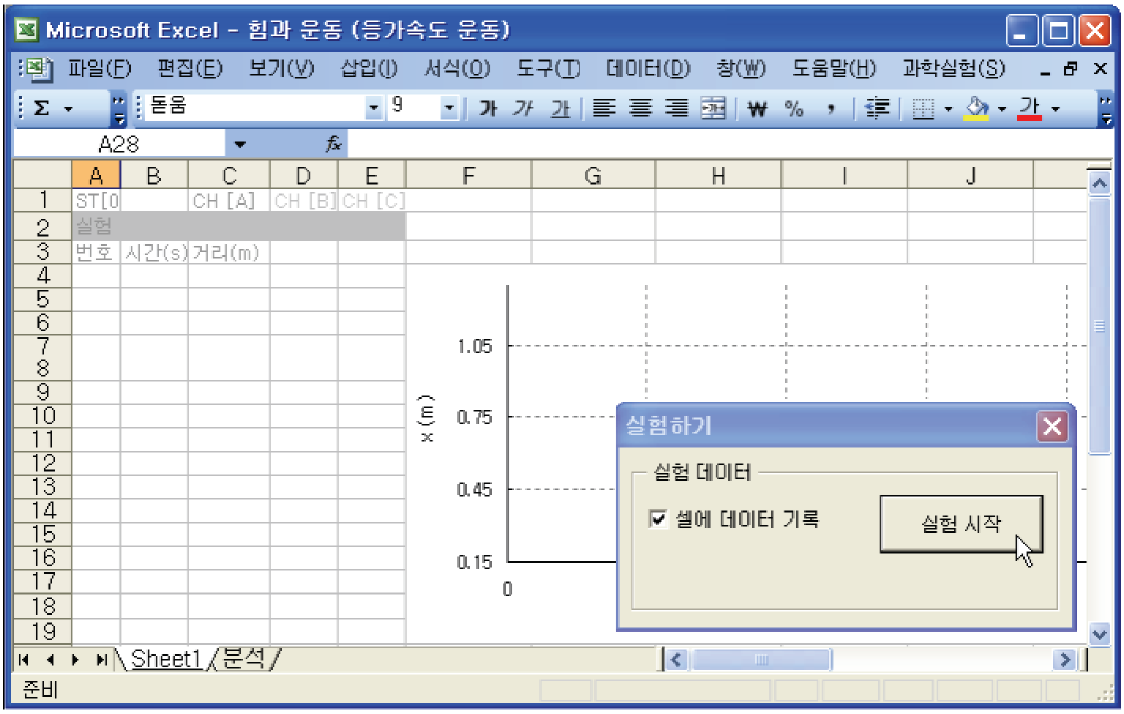
1. As in picture 3.5.5, set up the motion sensor at the right end of the track, and the pulley[93] at the left end, and connect the pendulum and the cart with a cord.

[93] The moment of inertia of the pulley used in this experiment is $3.09 \times 10^{-6} kg \cdot m^2$.

2. Open "Force and Motion(Uniformly Accelerated Motion).xls" file and execute the experiment.

a. Open "Sheet1" in Excel, then open [Science Cube]-[Experiment Setting] in the worksheet menu to set up the estimating interval as 0.1 second and the experiment time as 3.5 seconds.

b. As in picture 3.5.6, open [Science Cube]-[Experiment] window in worksheet menu of "Sheet1".




Picture 3.5.6 experiment in "Sheet1": If you click [Start Experiment] button, data will be collected within the sheet.

c. Put the cart 0.2m ahead of the motion sensor and hold it tight with one hand[94].

d. Click [Start Experiment] button. If you click it, the experiment data will be collected into "Sheet1".

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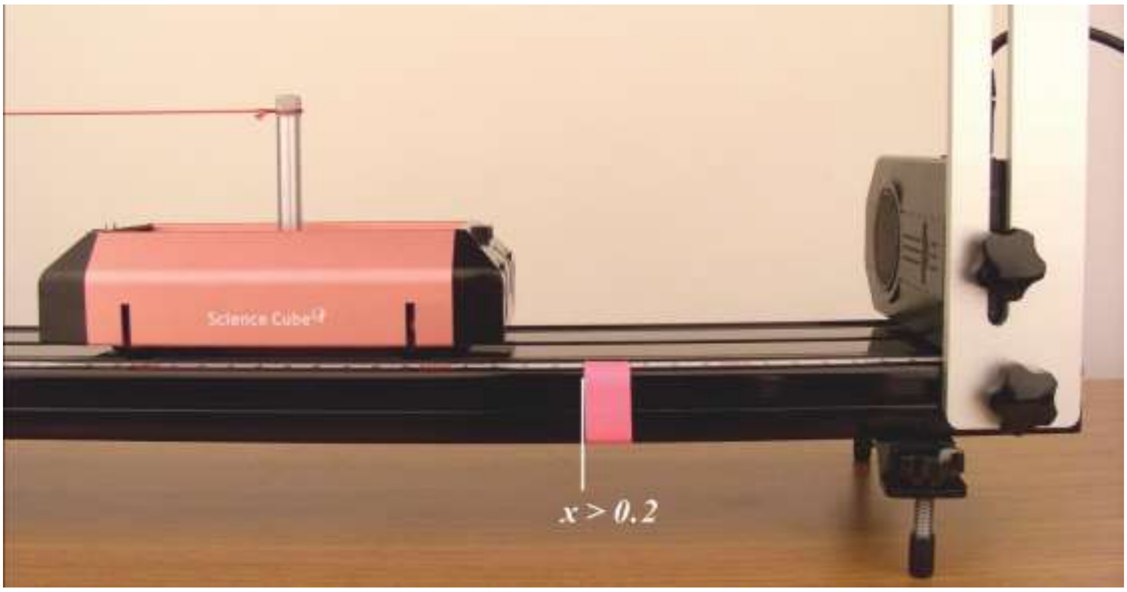
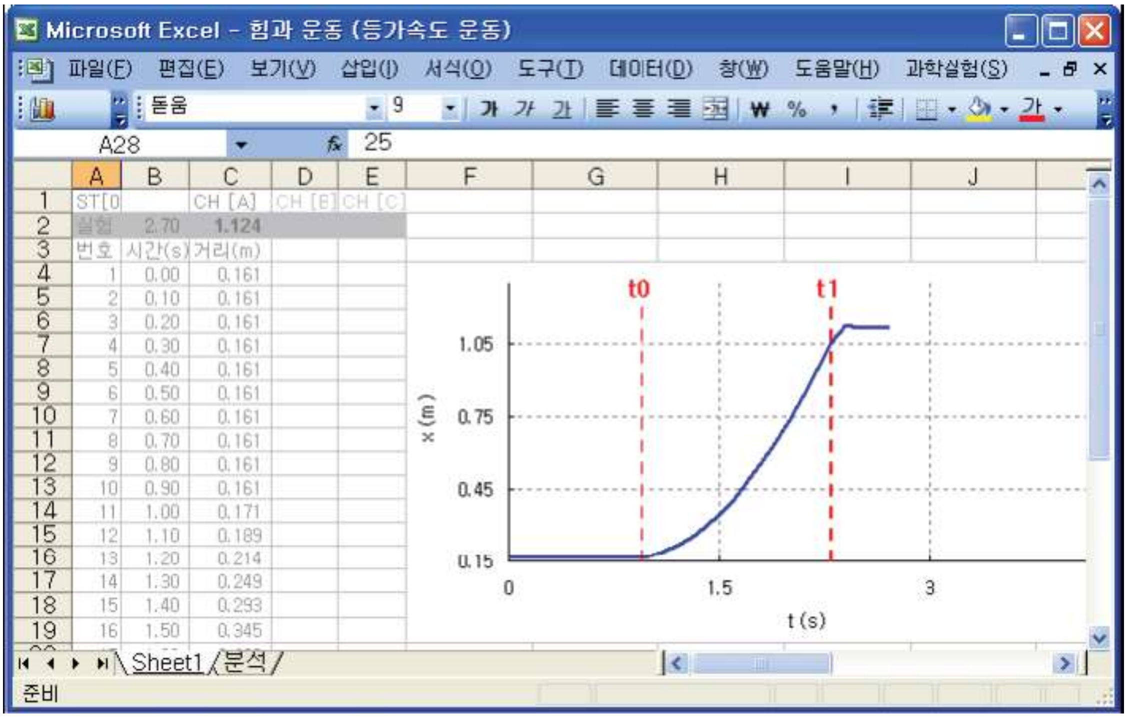
[94] You can attach the cart's side to the track's side with the holding tape. Detach the tape when the experiment gets started.




Picture 3.5.7 experiment by putting the cart 0.2m ahead of the motion sensor[95]

3. When the cart stops moving, click [Stop Experiment] button in [Experiment] window to stop collecting data and analyze the experiment.

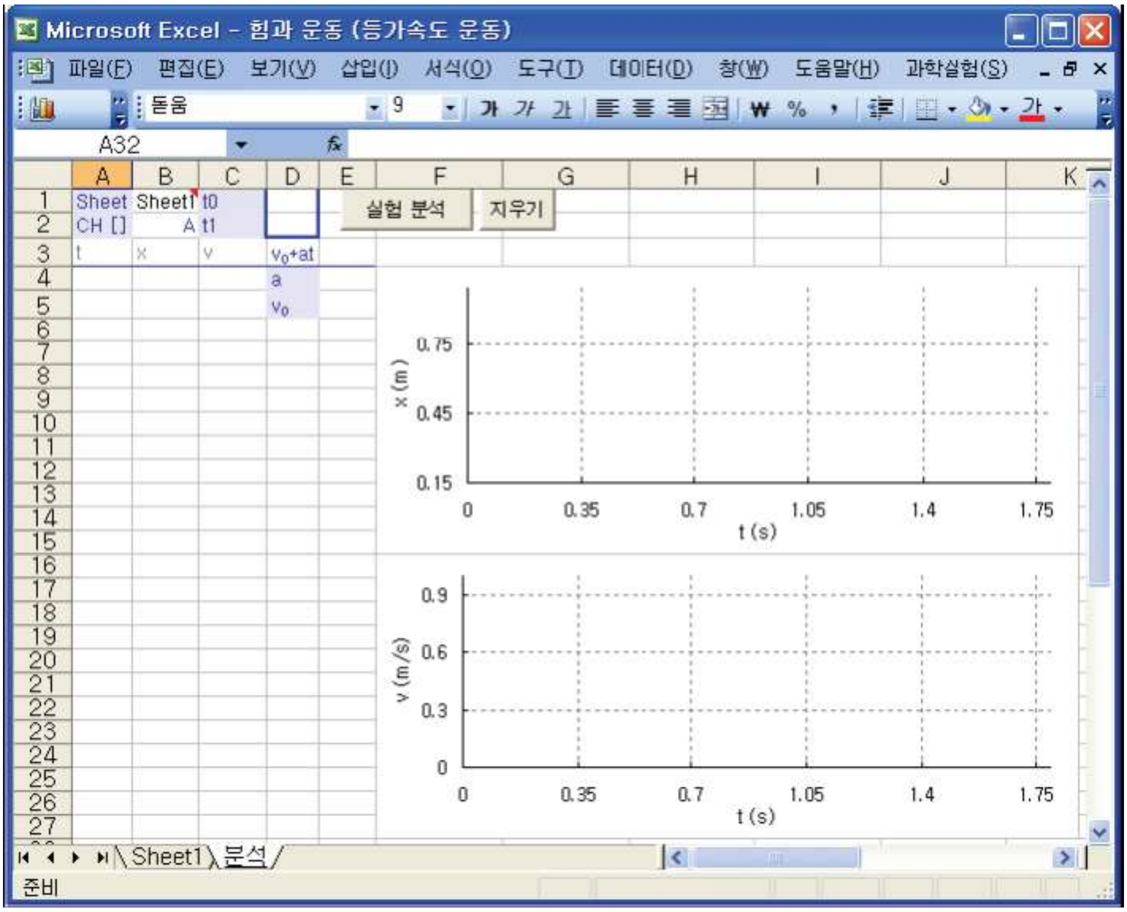



Picture 3.5.8 reading the time section that will be analyzed in “Sheet1”: You should read the time section t0 and t1 in “Sheet1”.

[95] Execute the experiment within the motion sensor’s sensing range 0.15-6.0m. This value can be different according to the motion sensor’s type. The starting point is indicated in the picture

a. As in picture 3.5.8, read the time value(x axis) of the time section t0 and t1 in "Sheet1", in which the cart do the uniform motion.

b. Open "Analysis" sheet and input the time t0 in cell D1 and the time t2 in cell D2. Click [Experiment Analysis] button and analyze the experiment data.




Picture 3.5.9 analyzing in "Analysis" sheet: If you click [Experiment Analysis] button, the velocity and the acceleration[96] are analyzed.

4. Change the pendulum's mass and the cart's mass and execute the experiment process above again.

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[96] The analyzed value of the acceleration $a$ is recorded in cell E4, and that of the initial velocity $v_0$ is recorded in E5. $v_0$ corresponds to the Y axis' intercept of the trend line's formula$(at+v_0)$ in $v-t$ graph.

Experiment Explanation:

1. Calculate the cart's acceleration with the experiment when the cart's mass is 0.525kg and the pendulum's mass is 0.05kg, and calculate the coefficient of friction between the cart and the track.

2. Compare acceleration a(T) of the theoretical value and of the experiment result within the experiment error range. Record the result of analysis in the table 3.5.2 below when the cart's mass M is 0.5kg and 1.0 kg, and when the pendulum's mass m is 0.05kg and 0.1kg.

| m | M | v0 | a(L) | $\pm\delta a$(L) | a(T) |
|---|---|---|---|---|---|
| 0.05 | 0.5 | | | | |
| 0.05 | 1.0 | | | | |
| 0.1 | 0.5 | | | | |

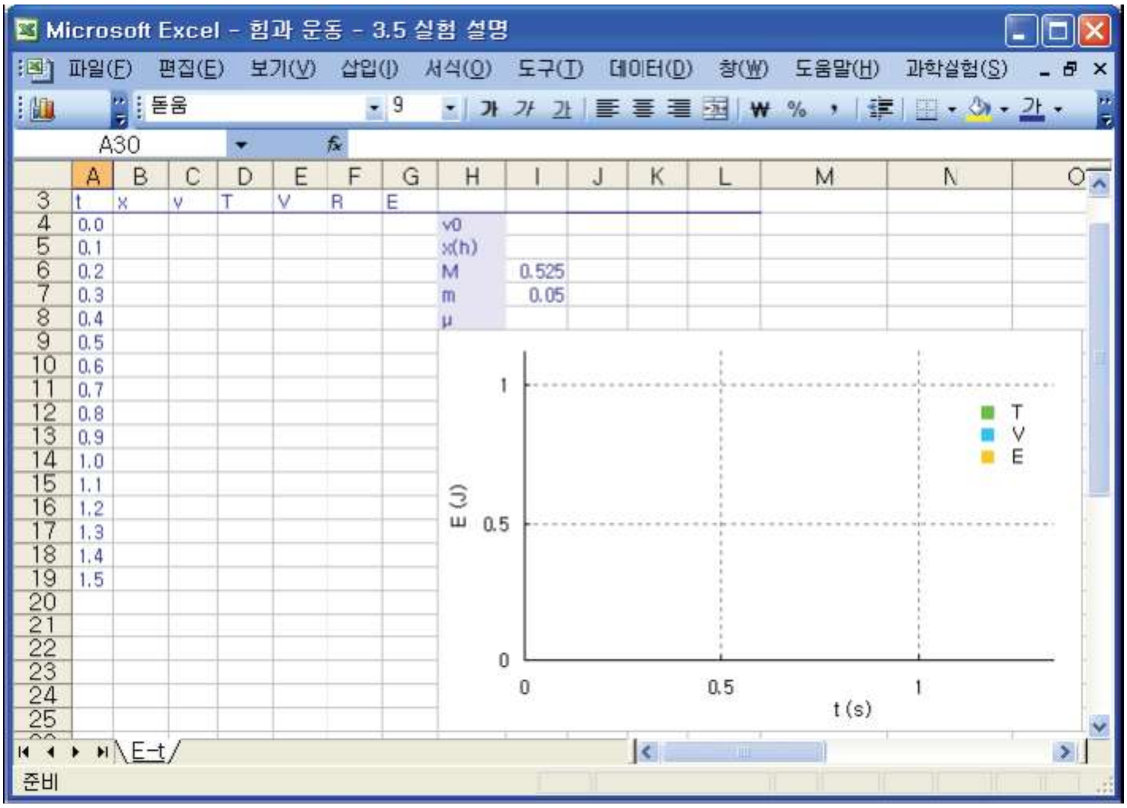
Table 3.5.2 result of the cart's motion according to the pendulum's falling

3. Draw the relationship graph between the cart's potential energy V, kinetic energy T, loss energy by the friction (R) and the total energy E[97], and see if the cart's total energy is preserved.

| t(s) | T | V | (R) | E |
|---|---|---|---|---|
| | | | | |

Table 3.5.3 analysis result of the cart's energy according to time

[97] Using the result analyzed in "Analysis" sheet, as picture 3.5.10, record in "E-t" sheet the time t in column A, the location x in column B and the velocity v in column C. Then apply the formula to the value of x and v and record the kinetic energy T, potential energy V and total energy E in column D, E, and G




Picture 3.5.10 drawing relationship graphs of energy and time T-t, V-t, and E-t[98]

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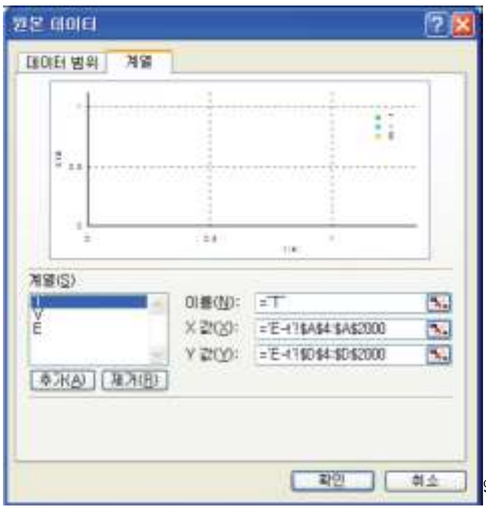



[98] Draw graphs of T-t, V-t, and E-t in one chart using the chart magician of Excel. Record the graphs original data series in column D, E and G after calculating it with formula. The picture above shows the process of setting up the [Original Data] series to add graphs of T, V, and E in the chart. In the X value(X), set up the range of time t's data series as $A$4:$A$2000. And the data range of Y value(Y) should be set up as column D, E, and G in which the energy data of T, V and E are recorded. For example, it should be set up as $D$4:$D$2000.

### 3.5.4 Experiment: Motion in Which the Force Changes According to Velocity

#### Experiment Prediction:

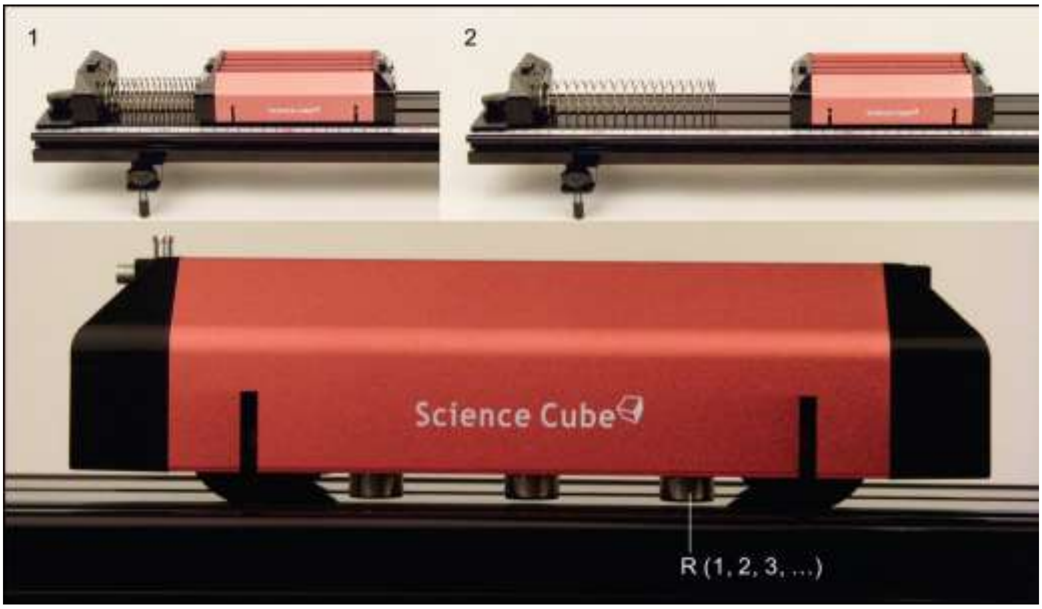
1. As in picture 3.5.11, think of a motion of a cart's that gets resistance by magnets.

   a. Guess how the cart's moving distance will be changed according to the resistance. How will the force the cart gets be changed according to the cart's velocity.

   b. Guess how the cart's energy loss rate will be different according to the number of magnets.

   c. What kind of motion does the cart show according to the initial velocity $v_0$?

#### Experiment Process: Cart's Motion According to Damping Resistance

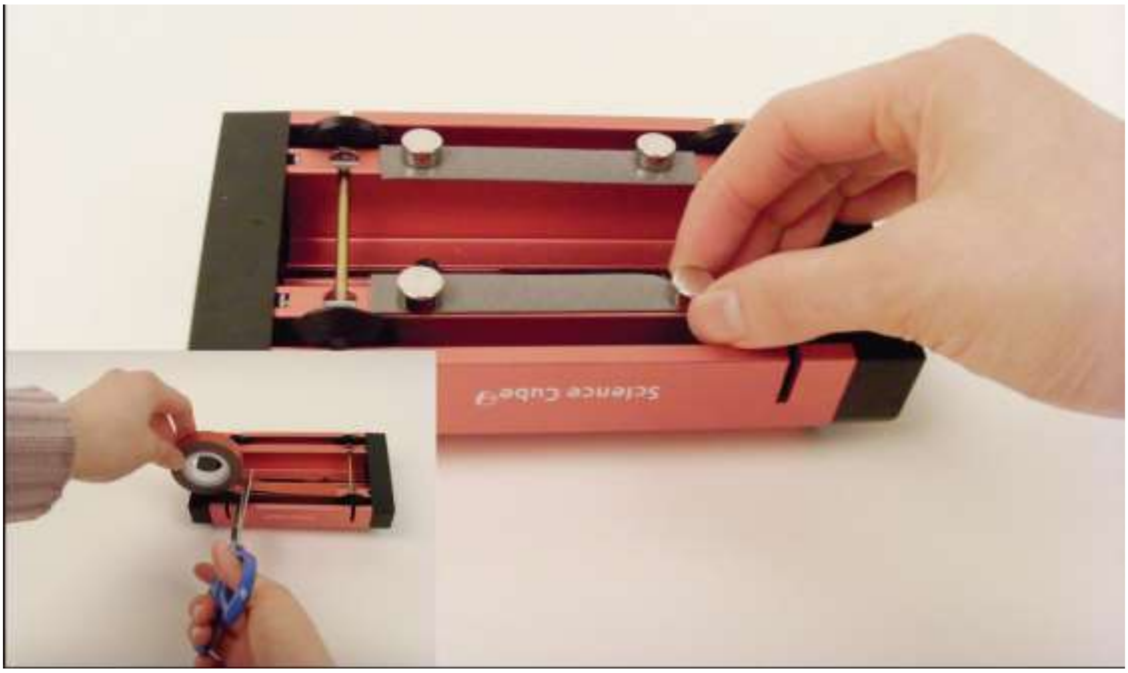



Picture 3.5.11 cart's motion that gets force which changes according to the velocity because of magnet[99]

[99] The sizes of resistance are expressed as R1, R2, R3 and so on. Execute the

1. Set up the bumper at the left end of the track and put the spring on the track. Set up the motion sensor at the right end of the track.

2. As in picture 3.5.12, attach the magnet to the bottom of the cart with the double stick tape.



Picture 3.5.12 making resistance by attaching magnets[100] to the cart

3. Open “Force and Motion(HMLR[101]).xls” file and execute the experiment.

a. Open “Sheet1” and open [Science Cube]-[Experiment Setting] window to set up the estimating interval as 0.1 second, and the experiment time as 15 seconds.

b. As in picture 3.5.13, open [Science Cube]-[Experiment] window in the worksheet menu of “Sheet1” and click [Start Experiment] button.

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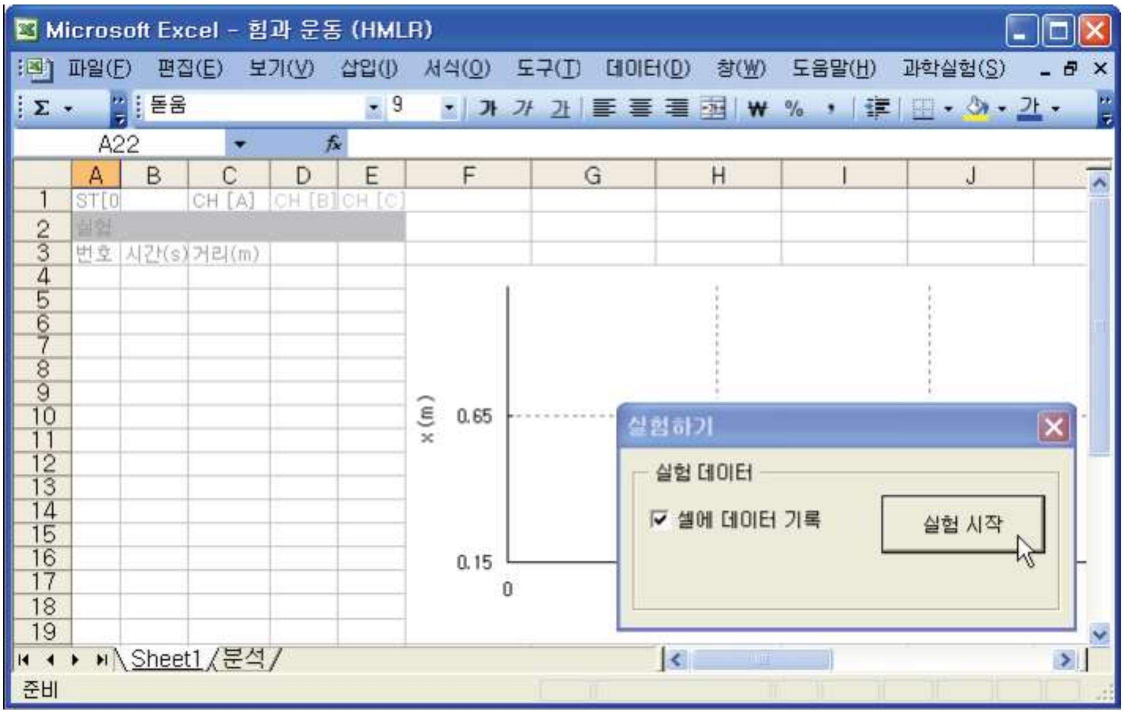
experiment to see how the resistance changes according to the number of magnets.

[100] Use neodymium magnets of proper thickness so that the magnets do not touch the surface of the track.

[101] Horizontal Motion with Linear Resistance




Picture 3.5.13 experiment in “Sheet1”: If you click [Start Experiment] button, data will be collected within the sheet.

c. If you click [Start Experiment] button, the experiment data will be collected within the sheet as in the graph section (a)-(b) of picture 3.5.14.

d. While moving the cart slowly to the left and compressing the spring, data is collected as in the graph section (b)-(c) of picture 3.5.15. Section (c)-(d) is the state of staying the same after the cart has been moved[102].

e. If you release the cart slightly at (d) of picture 3.5.14, the cart is pushed according to the spring and starts moving.

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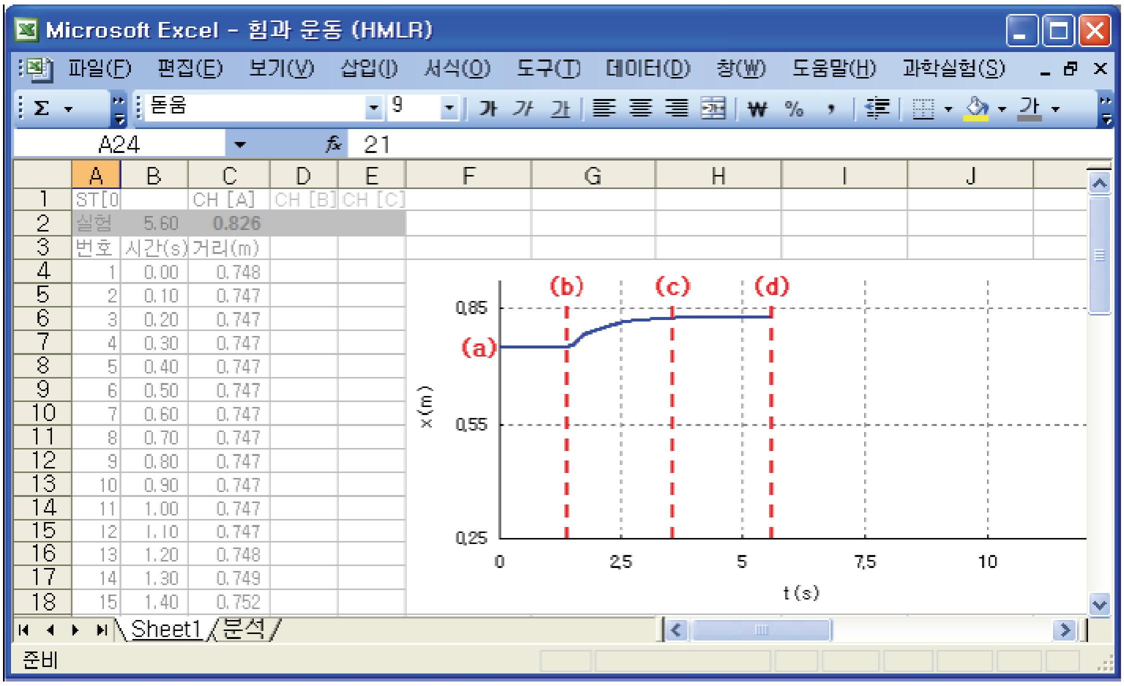
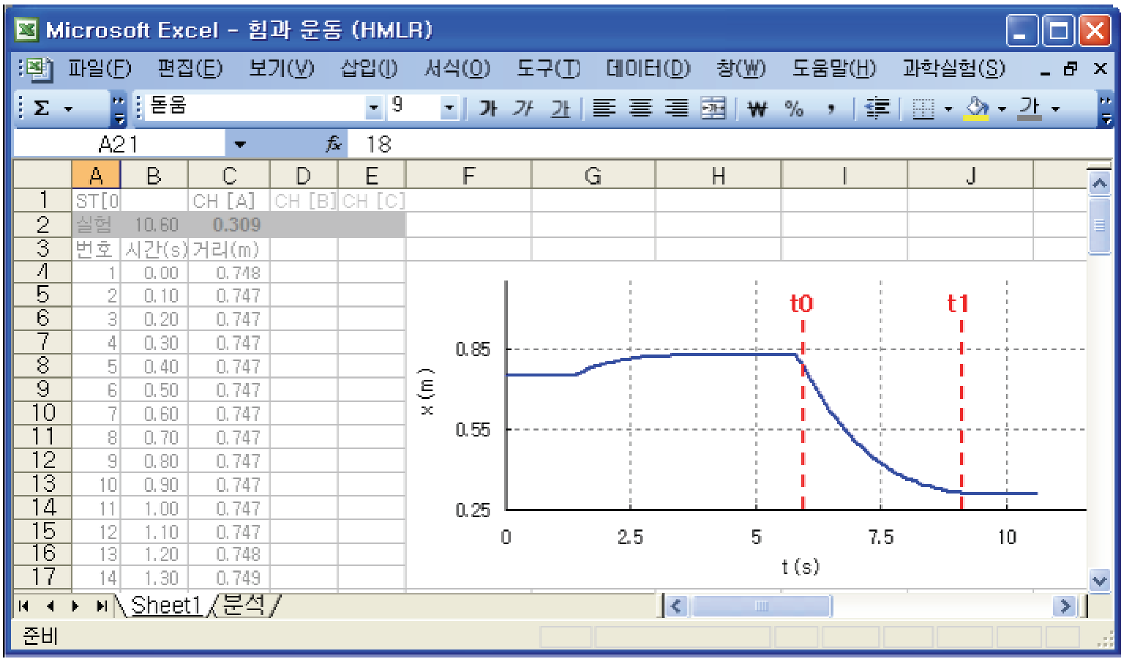
[102] The difference between section(c)-(d) and section (a)-(b) is the spring's compressed length. To change this length, change the initial velocity $v_0$ when the cart starts moving at (d).




Picture 3.5.14 starting experiment in "Sheet1" : If you release the cart at (d), the cart starts moving.

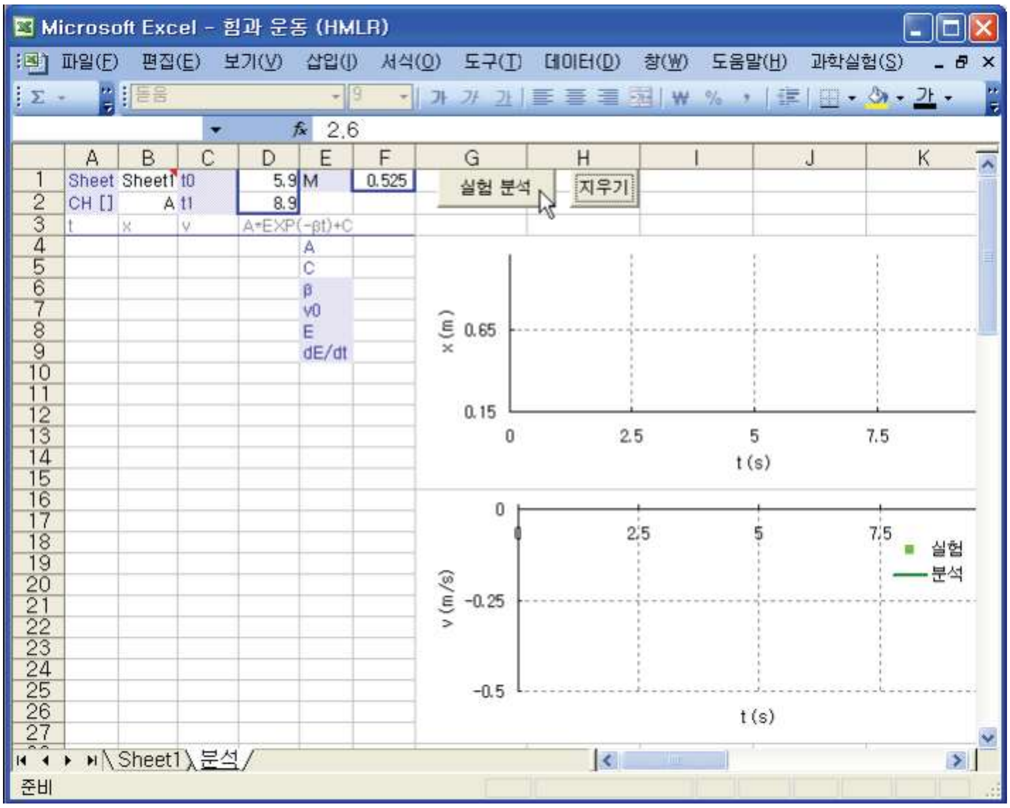
4. When the cart stops moving, click [Stop Experiment] in [Experiment] window to stop collecting data and analyze the experiment.




Picture 3.5.15 reading time section t0 and t1 that will be analyzed in "Sheet1"

a. As in picture 3.5.15, read the time value(x axis) of the time section t0 and t1 in “Sheet1”, in which the cart is in motion.

b. Open “Analysis” sheet to input the value of the time t0 to cell D1 and that of the time t1 in cell D2. Click [Experiment Analysis] to analyze the experiment data.




Picture 3.5.16 analyzing in “Analysis” sheet: If you click [Experiment Analysis] button, the velocity and the acceleration will be analyzed.

6. Execute the experiment by changing the number of magnets[103] and the cart’s initial velocity $v_0$.

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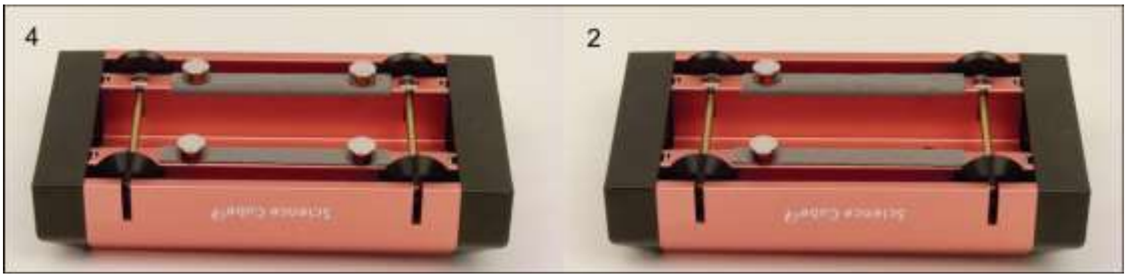
103



Experiment Explanation:

1. Change the spring's compressed length $\Delta x$ and the number of magnets N to do the experiment and fill out table 3.5.4[104].

| $\Delta x$ | N | $v_0$ | $a_0$ | $x$ | $dE/dt$ | $v(t)$[105] |
|---|---|---|---|---|---|---|
| 0.04<br>0.06<br>0.08 | 2 | | | | | |
| 0.06 | 4<br>6 | | | | | |

Table 3.5.4 result of cart's motion that gets force in proportion to velocity[105]

2. With the result of table 3.5.4, explain the relationship between the velocity's damping size[106], the force's action and the energy.

3. When $\Delta x = 0.08$, change the number of magnets as 2, 4, and 6 and execute the experiment. From the result, combine and draw each $v-t$ graph in one chart and explain it.

4. Draw the graph of the kinetic energy according to time T-t and the graph of energy loss rate (dE/dt)-t and explain them.

[104] Those are the velocity $v_0$ and the acceleration $a_0$ when the time $t_0 = 0$, the velocity's formula $v(t)$ at time $t$, the cart's moving distance $x$, the damping constant $\beta$ and the energy loss rate $dE/dt$.

[105] $v(t) = -0.383e^{-0.490t}$

[106] When the object gets the force of $F = -bv$ on the horizontal surface, the damping constant should be calculated as $\beta = b/m$.

### 3.5.5 Exercise

1. Table 3.5.5 is the result of the cart's motion on the slope. The acceleration $a_{Lab}$ is an experimental value. Express the acceleration of gravity $g_{Lab} \pm \delta g$ within the error range and explain the cart's acceleration $a_{Lab}$ by comparing it to the theoretical value $a_r$.

| | $a_{Lab}$ | $g_{Lab} \pm \delta g$ | $a_T$ |
|---|---|---|---|
| 3.0±0.2° | 0.506 | | |
| 5.0±0.2° | 0.849 | | |

Table 3.5.5 result of cart's motion according to the slope's angle

2. Table 3.5.6 is the analysis result of the cart's motion according to the pendulum's falling. When the kinetic energy according to time t is T, the potential energy is V, the energy loss according to the friction is R and the total energy is E, draw graphs of T-t, V-t, and E-t and explain if the mechanical energy is conserved.

| t(s) | x(m) | v(m/s) | T | V | R | E |
|---|---|---|---|---|---|---|
| 0.5 | 0.332 | 0.400 | | | 0.024 | |
| 0.6 | 0.380 | 0.480 | | | 0.027 | |
| 0.7 | 0.437 | 0.570 | | | 0.031 | |
| 0.8 | 0.499 | 0.620 | | | 0.036 | |
| 0.9 | 0.568 | 0.690 | | | 0.041 | |
| 1.0 | 0.645 | 0.770 | | | 0.046 | |
| 1.1 | 0.728 | 0.830 | | | 0.052 | |
| 1.2 | 0.818 | 0.900 | | | 0.059 | |

Table 3.5.6 analysis result of the cart's motion according to the pendulum's falling

3. Table 3.5.7 is the result of the damping motion of a cart with 0.525kg mass when the resistance by the magnet is R2 and R4. And Picture 3.5.17 is the graph of the result above.

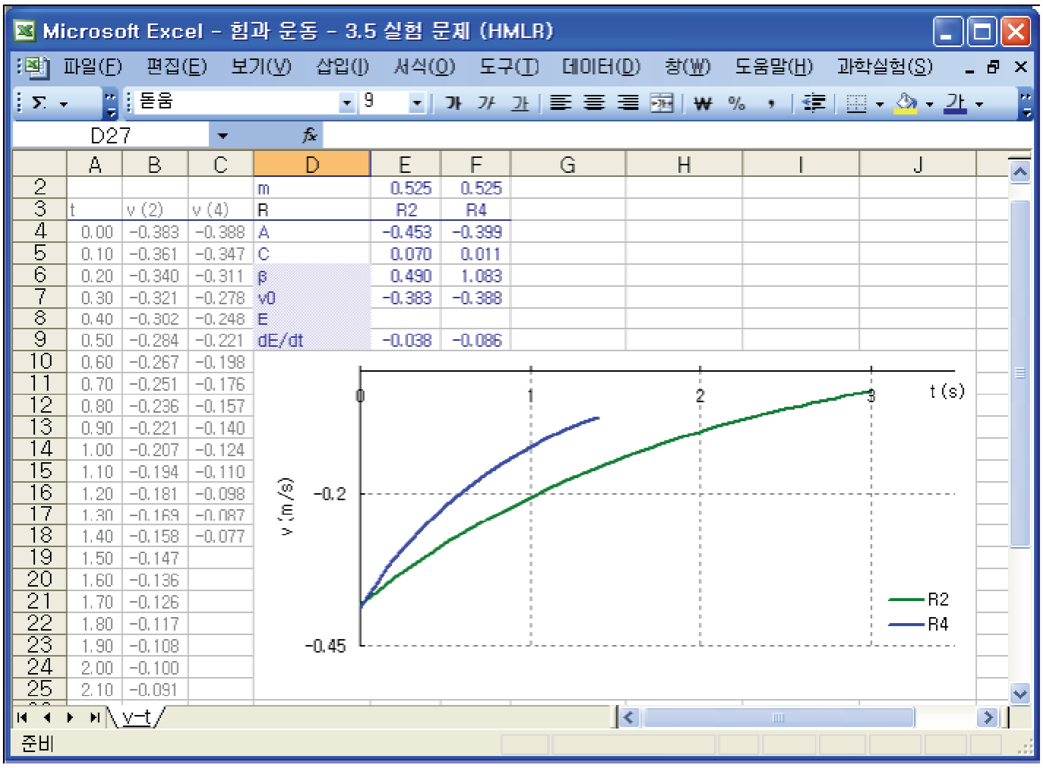
a. Complete the formula of $v(t)$ in the table and draw the energy graph E-t.

b. Calculate the cart's acceleration when the time $t_0 = 0$.

c. Explain the cart's motion from table 3.5.7 and picture 3.5.17.

| | $v_0$ | $\beta$ | $dE/dt$ | $v(t)$ |
|---|---|---|---|---|
| R2 | 0.506 | 0.490 | -0.038 | |
| R4 | 0.849 | 1.083 | -0.086 | |

Table 3.5.7 result of a cart's motion that gets the force in proportion to the velocity




| | A | B | C | D | E | F |
|---|---|---|---|---|---|---|
| 2 | | | | m | 0.525 | 0.525 |
| 3 | t | v (2) | v (4) | R | R2 | R4 |
| 4 | 0.00 | -0.383 | -0.388 | A | -0.453 | -0.399 |
| 5 | 0.10 | -0.361 | -0.347 | C | 0.070 | 0.011 |
| 6 | 0.20 | -0.340 | -0.311 | β | 0.490 | 1.083 |
| 7 | 0.30 | -0.321 | -0.278 | v0 | -0.383 | -0.388 |
| 8 | 0.40 | -0.302 | -0.248 | E | | |
| 9 | 0.50 | -0.284 | -0.221 | dE/dt | -0.038 | -0.086 |
| 10 | 0.60 | -0.267 | -0.198 | | | |
| 11 | 0.70 | -0.251 | -0.176 | | | |
| 12 | 0.80 | -0.236 | -0.157 | | | |
| 13 | 0.90 | -0.221 | -0.140 | | | |
| 14 | 1.00 | -0.207 | -0.124 | | | |
| 15 | 1.10 | -0.194 | -0.110 | | | |
| 16 | 1.20 | -0.181 | -0.098 | | | |
| 17 | 1.30 | -0.169 | -0.087 | | | |
| 18 | 1.40 | -0.158 | -0.077 | | | |
| 19 | 1.50 | -0.147 | | | | |
| 20 | 1.60 | -0.136 | | | | |
| 21 | 1.70 | -0.126 | | | | |
| 22 | 1.80 | -0.117 | | | | |
| 23 | 1.90 | -0.108 | | | | |
| 24 | 2.00 | -0.100 | | | | |
| 25 | 2.10 | -0.091 | | | | |

Picture 3.5.17 $v-t$ graph of a cart's motion that gets the force in proportion to the velocity